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# On degree-distance index of fuzzy Mycielskian graph and its partial complement

**Neeta M. Chougule** *, **D. B. Unde** **

**Department of Mathematics, KIT'S College of Engineering, Kolhapur*

***Department of Mathematics, DR. J.J.Magdum College of Engineering, Jaysingpur*

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**ABSTRACT**

Let $G = (V, \sigma, \mu)$ be a connected fuzzy graph with $n$-vertices and $m$-edges. The degree sum of any two arbitrary vertices $u$ and $v$ in a fuzzy graph $G$ is defined as $\sigma(u)d(u) + \sigma(v)d(v)$. Then the degree-distance index $DD^f(G)$ of a fuzzy graph $G$ is defined as the sum of the distance between every pair of vertices together with their degree sum. In this paper, the degree-distance index $DD^f(G)$ of a connected fuzzy graph is introduced and we investigate $DD^f(G)$ of fuzzy Mycielskian graph and partial complement of fuzzy Mycielskian graph of a connected fuzzy graph $G$.

**Keywords**: Fuzzy graph, Fuzzy Mycielskian graph, Degree-distance index.

**Subject Classification:** 05C09, 05C72.

## 1 Introduction

Topological indices are molecular descriptors widely used in areas like mathematical chemistry, molecular topology and chemical graph theory. Such numerical parameters are representatives of molecular compounds that characterize the topology of the corresponding molecular graph. Molecular properties are correlated with the chemical structure of the compound. In a molecular graph, vertices and edges are representatives of atoms and bonds respectively. The first investigation into Wiener index was made by Harold Wiener in 1947 [13], during a study of boiling point of paraffins. It was chemists who used Wiener index decades before it captivated the attention of mathematicians. Innovative results connected to Wiener index were reported during the middle of 1970's and this gradually reaped great esteem. Wiener index is a topological index, which has been studied both from theoretical and application point of view. Wiener index of graphs have been studied in the field of Mathematics, Chemistry, Physics and Molecular Biology [4, 7, 6, 13].

Inspired by Zadeh's revolutionary fuzzy set theory [15], Rosenfeld [11] set forth the concept of a fuzzy graph in 1975. Meanwhile, Yeh and Bang [14] also studied fuzzy graphs independently and provided some of its applications in clustering analysis. Reference [11] provides the basics of fuzzy relations, blocks, fuzzy bridges and fuzzy graph distances. Fuzzy analogues of several graph theoretic concepts like line graphs, automorphism of graphs, interval graphs, etc. can be seen in [1, 2, 8, 9, 14].

Basic definitions and concepts of fuzzy graphs are given below. Undefined terminology in this paper may be found in [10].

Throughout this paper, we consider the simple fuzzy graphs. That is a fuzzy graph without multiple edges or loops. Let $G = (V, \sigma, \mu)$ be a simple fuzzy graph with $n$ vertices and $m$ edges. The membership values of the vertices $\{v_1, v_2, v_3, \cdots, v_n\}$ and edges $\{e_1, e_2, e_3, \cdots, e_m\}$ of a fuzzy graph $G$ are $\{\sigma(v_1), \sigma(v_2), \sigma(v_3), \cdots, \sigma(v_n)\}$ and $\{\mu(e_1), \mu(e_2) \quad, \mu(e_3), \cdots, \mu(e_m)\}$ respectively. The degree of a vertex $v \in V(G)$ is denoted by $d_G(v)$ and is defined as the sum of the membership values of the edges incident to a vertex $v \in V(G)$. The complement of a fuzzy graph $G = (V, \sigma, \mu)$ is denoted by $\overline{G} = (V, \overline{\sigma}, \overline{\mu})$, any two vertices $v_i, v_j \in V(\overline{G})$ are adjacent if and only if $v_i$ and $v_j$ are non-adjacent vertices in $G$ and vice-versa.

## 1.1 Preliminaries

**Definition 1[12]**. *Let $G = (V, \sigma, \mu)$ be a connected fuzzy graph. For any path $P: u_0, u_1, u_2, \cdots, u_n$ the length of $P$ is defined as:*

$$L(P) = \sum_{i=1}^{n} \mu(u_{i-1}, u_i).$$

In other words, the length of the path between the vertices $u$ and $v$ in a fuzzy graph $G$ is the sum of the membership values of the edges involved in $u - v$ path and length of shortest path is denoted by $L_s(P)$.

**Definition 2** *Let $G = (V, \sigma, \mu)$ be a connected fuzzy graph. For any pair of vertices $u, v \in V(G)$, the distance is denoted by $d(u, v)$ and is defined as the minimum length of shortest path among all possible u-v shortest paths.*



Vol. 21, No. 1, (2024)
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$$d(u,v) = \wedge \{L_s(P_i) : i = 1,2,3,\cdots\}.$$

**Definition 3** *Let* $G = (V, \sigma, \mu)$ *be a fuzzy graph. Then the complement of a fuzzy graph* $\overline{G} = (V, \overline{\sigma}, \overline{\mu})$ *is defined in such a way that for* $u, v \in V(\overline{G})$ *are adjacent if and only if* $u$ *and* $v$ *are non-adjacent vertices in* $G$ *and vice-versa. The membership values of vertices and edges of* $\overline{G}$ *are given as follows:*

$$\overline{\sigma}(u) = \sigma(u)$$
$$\overline{\mu}(uv) = \wedge [\sigma(u), \sigma(v)].$$

**Definition 4** *Let* $G = (V, \sigma, \mu)$ *be a fuzzy graph with vertex set* $V(G) = \{v_1, v_2, v_3, \cdots, v_n\}$ *and edge set* $E(G) = \{e_1, e_2, e_3, \cdots, e_m\}$ *with membership values* $\{\sigma(v_1), \sigma(v_2), \sigma(v_3), \cdots, \sigma(v_n)\}$ *and* $\{\mu(e_1), \mu(e_2), \mu(e_3), \cdots, \mu(e_m)\}$ *of vertices and edges respectively. The fuzzy Mycielskian graph* $M(G)$ *of* $G$ *is the graph with* $V(M(G)) = V(G) \cup X(G) \cup \{x\}$ *where* $X(G) = \{x_1, x_2, x_3, \cdots, x_n\}$ *are the vertices corresponding to each* $v_i \in V(G)$. *Then* $E(M(G)) = E(G) \cup \{v_i x_j : v_i v_j \in E(G)\} \cup \{x x_i : 1 \leq i \leq n\}$., *where* $i \neq j$ *The membership values for each vertex and each edge in* $M(G)$ *is assigned as follows:*

- $\sigma_{M(G)}(u) = \sigma_G(u)$ where $u \in V(G)$
- $\sigma_{M(G)}(u) = \sigma_G(v)$ where $u \in X(G)$ and $u$ is the corresponding vertex of $v$ in $G$
- $\sigma_{M(G)}(u) = 1$ where $u = x$
- $\mu_{M(G)}(uv) = \mu_G(uv)$ where $uv \in E(G)$
- $\mu_{M(G)}(uv) = \mu_G(v_i v_j)$ where $uv = v_i x_j$ and $x_j$ is the vertex corresponding vertex $v_j$ in $G$.
- $\mu_{M(G)}(uv) = \sigma_G(v_i)$ where $uv = x x_i$ and $x_i$ is the vertex corresponding vertex $v_i$ in $G$.

A fuzzy graph $G$ and its fuzzy Mycielskian graph $M(G)$ is depicted in figure 1.

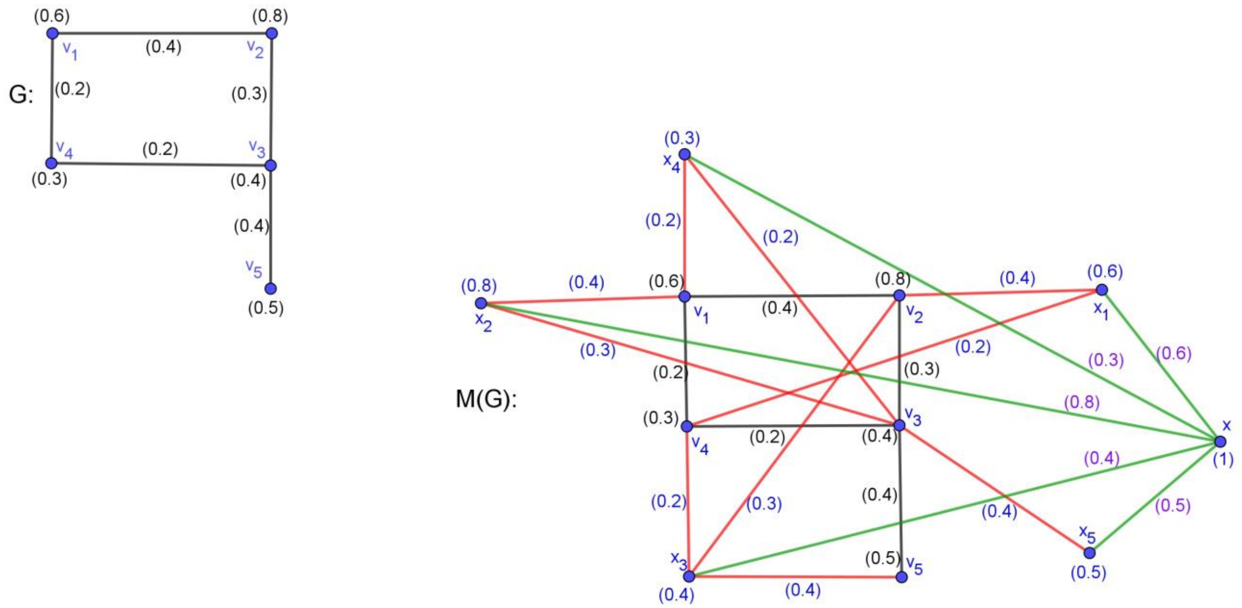


Figure 1. A fuzzy graph $G$ and its fuzzy Mycielskian graph $M(G)$.

**Definition 5** *Let $G = (V, \sigma, \mu)$ be a fuzzy graph with vertex set $V(G) = \{v_1, v_2, v_3, \cdots, v_n\}$ and edge set $E(G) = \{e_1, e_2, e_3, \cdots, e_m\}$ with membership values $\{\sigma(v_1), \sigma(v_2), \sigma(v_3), \cdots, \sigma(v_n)\}$ and $\{\mu(e_1), \mu(e_2), \mu(e_3), \cdots, \mu(e_m)\}$ of vertices and edges respectively. The partial complement of a fuzzy Mycielskian graph $M^*(G)$ of $G$ is the graph with $V(M^*(G)) = V(G) \cup X(G) \cup \{x\}$ where $X(G) = \{x_1, x_2, x_3, \cdots, x_n\}$ are the vertices corresponding to each $v_i \in V(G)$. Then $E(M^*(G)) = E(\overline{G}) \cup \{v_i x_j : v_i v_j \notin E(G)\} \cup \{x x_i : 1 \leq i \leq n\}. i \neq j$ The membership values for each vertex and each edge in $M^*(G)$ is assigned as follows:*

- $\sigma_{M^*(G)}(u) = \sigma_G(u)$ where $u \in V(G)$
- $\sigma_{M^*(G)}(u) = \sigma_G(v)$ where $u \in X(G)$ and $u$ is the corresponding vertex of $v$ in $G$
- $\sigma_{M^*(G)}(u) = 1$ where $u = x$
- $\mu_{M^*(G)}(uv) = \wedge [\sigma(u), \sigma(v)]$ where $uv \notin E(G)$
- $\mu_{M^*(G)}(uv) = \wedge [\sigma(u), \sigma(v)]$ where $uv = v_i x_j$ and $v_i v_j \notin E(G)$ and $x_j$ is the vertex corresponding to $v_j$ in $G$
- $\mu_{M^*(G)}(uv) = \sigma_G(v_i)$ where $uv = x x_i$ and $x_i$ is the vertex corresponding to $v_i$ in $G$

A fuzzy graph $G$ and its partial complement of fuzzy Mycielskian graph $M^*(G)$ is depicted in figure 2.

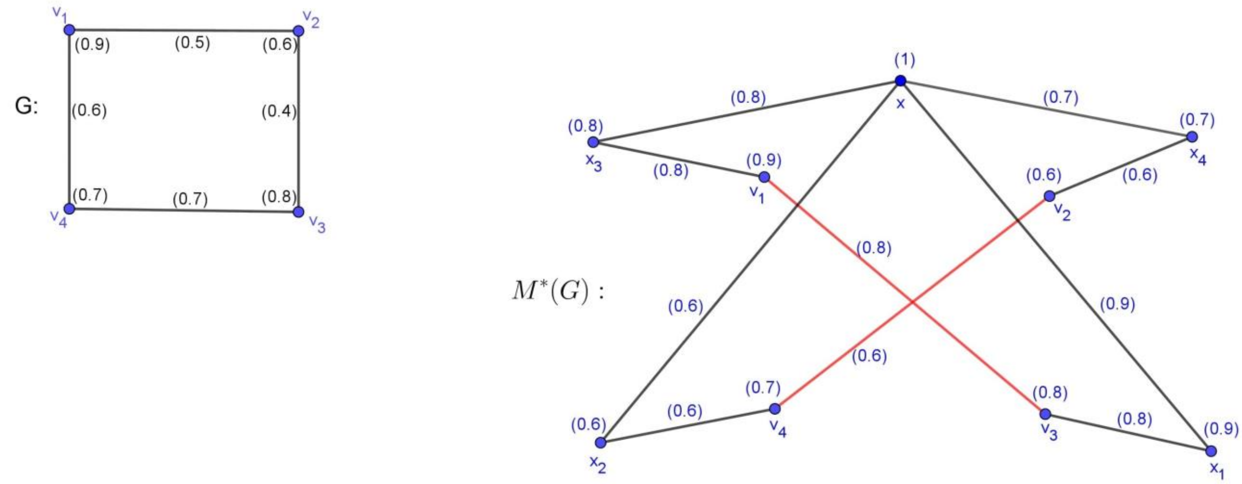


Figure 2. A fuzzy graph $G$ and its partial complement of fuzzy Mycielskian graph $M^*(G)$.

## 1.2 Motivation

Since, the distance based topological indices are most difficult to study due to uncertainty of shortest paths between any pair of vertices at more than distance 3. Therefore, the distance-based topological indices have got less attention by the researchers. For fuzzy graphs, only few distance based topological indices have been studied such as Wiener index [3]. The degree-distance index of crisp graphs put forwarded in [5] and it is defined as:

$$DD(G) = \sum_{\{u,v\} \subseteq V(G)} d(u,v)[d_G(u) + d_G(v)].$$

Motivated by the definition of degree-distance index of crisp graphs, we have defined the degree-distance index of fuzzy graphs. The definition is given in the next section.

# 2 Results

**Definition 6** *The degree-distance index* $DD^f(G)$ *of a connected fuzzy graph* $G$ *is defined as follows:*

$$DD^f(G) = \frac{1}{2} \sum_{\{u,v\} \subseteq V(G)} d(u,v)[\sigma(u)d(u) + \sigma(v)d(v)].$$

**Observation 1** *Let* $G = (V, \sigma, \mu)$ *be a fuzzy graph and* $M(G)$ *is its Mycielskian fuzzy graph. Then*



Vol. 21, No. 1, (2024)
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- $deg_{M(G)}(v) = \sum_{i=1}^{n} \sigma(x_i)$ where $v = x$
- $deg_{M(G)}(v) = 2deg_G(v_i)$ where $v = v_i$
- $deg_{M(G)}(v) = deg_G(v_i) + \sigma_G(v_i)$ where $v = x_i$

**Observation 2** *Let $M(G)$ be a fuzzy Mycielskian graph of the fuzzy graph $G = (V, \sigma, \mu)$. Then the distance between every pair of vertices in $M(G)$ are given below:*

- $d_{M(G)}(u, v) = \sigma(x_i)$ where $u = x$ and $v = x_i$
- $d_{M(G)}(u, v) = \wedge [\mu(xx_i) + \mu(x_i v_j)]$ where $u = x$ and $v = v_j$ and $i \neq j$
- $d_{M(G)}(u, v) = [\mu(x_i, x) + \mu(x, x_j)]$ or $\wedge [\mu(x_i, v_k) + \mu(v_k, x_j)]$ where $u = x_i$ and $v = x_j$
- $d_{M(G)}(u, v) = \mu(uv)$ where $u = v_i$ and $v = v_j$ and $v_i$ and $v_j$ are adjacent in $G$
- $d_{M(G)}(u, v) = \wedge [L_s(P_i): i = 1,2,3, \cdots]$ where $u = v_i$ and $v = v_j$ and $v_i$ and $v_j$ are nonadjacent in $G$
- $d_{M(G)}(u, v) = \mu(uv)$ where $u = v_i$ and $v = x_j$ where $x_j$ is the corresponding vertex of $v_j$ such that $v_i$ and $v_j$ are adjacent in $G$.
- $d_{M(G)}(u, v) = \wedge [\mu(v_i v_j) + \mu(v_j x_i)]$ if $u = v_i$ and $v = x_i$.
- $d_{M(G)}(u, v) \leq \wedge [\mu(v_i, x_j) + \mu(x_j, x) + \mu(x, x_k)]$ or $\wedge [\mu(v_i, v_j) + \mu(v_j, x_k)]$

where $u = v_i$ and $v = x_k$ where $x_k$ is the corresponding vertex of $v_k$ such that $v_i$ and $v_k$ are nonadjacent in $G$.

**Theorem 3** *Let $G = (V, \sigma, \mu)$ be a connected fuzzy graph with $n$-vertices, $m$-edges and maximum(minimum) degree $\Delta(\delta)$. Then the degree-distance index of fuzzy Mycielskian graph $M(G)$ of a fuzzy graph $G$ is given by*

$$DD^f(M(G)) \leq n[3n + 5\Delta + 2(n-1)(\Delta + 1) + 1] + (3\Delta + 1)[4n(n-1) + 2n] + 4\Delta(m + \chi(G)),$$

Where $\chi(G) = \sum_{\{v_i, v_j\} \subset V(M(G))} [\wedge (L_s(P_i: i = 1,2,3, \cdots))]$.

*Proof.* Regarding to the different possible cases which $u$ and $v$ can be chosen from the set $V(M(G))$ the following cases are considered:

**Case 1.** If $u = x$ and $v = x_i$, then the length of the shortest $uv$-path is given by $\sigma(x_i)$. Therefore, the degree-distance index $DD^f(M(G))$ of fuzzy Mycielskian graph is given by:

$$DD^f(M(G)) = \frac{1}{2} \sum_{\{u,v\} \subseteq V(M(G))} d_{M(G)}(u, v)[\sigma(u) d_{M(G)}(u) + \sigma(v) d_{M(G)}(v)].$$

Since $d(x, x_i) = d(x_i, x)$. Therefore, $DD^f(M(G))$ of fuzzy Mycielskian graph can be re-



Vol. 21, No. 1, (2024)
ISSN: 1005-0930

written as:

$$DD^f(M(G)) = \sum_{\{u,v\}\subseteq V(M(G))} d_{M(G)}(u,v)[\sigma(u)d_{M(G)}(u) + \sigma(v)d_{M(G)}(v)].$$

By Observations 1 and 2 we have,

$$DD^f(M(G)) = \sum_{\{x,x_i\}\subset V(M(G))} d_{M(G)}(x,x_i)[\sigma(x)d_{M(G)}(x) + \sigma(x_i)d_{M(G)}(x_i)]$$

$$= \sum_{\{x,x_i\}\subset V(M(G))} \sigma(x_i)[\sigma(x)[\sum_{i=1}^{n} \sigma(x_i)] + \sigma(x_i)[d_G(v) + \sigma_G(v)]]$$

Since, $0 \le \sigma(x_i) \le 1$, $0 \le \mu(v_i, v_j) \le 1$, $\sum_{i=1}^{n} \sigma(x_i) \le n$ and $\delta \le d(u) \le \Delta$. Therefore,

$$DD^f(M(G)) \le \sum_{\{x,x_i\}\subset V(M(G))} 1[1(n) + 1(\Delta + 1)]$$

$$\le n(n + \Delta + 1).$$

**Case 2.** If $u = x$ and $v = v_j$, then the degree-distance index $DD^f(M(G))$ of fuzzy Mycielskian graph is given by:

$$DD^f(M(G)) = \sum_{\{x,v_j\}\subset V(M(G))} d_{M(G)}(x,v_j)[\sigma(x)d_{M(G)}(x) + \sigma(v_j)d_{M(G)}(v_j)].$$

By Observations 1 and 2 we have,

$$DD^f(M(G)) = \sum_{\{x,v_j\}\subset V(M(G))} \wedge [\mu(x,x_i) + \mu(x_i,v_j)][\sigma(x)d_{M(G)}(x) + \sigma(v_j)d_{M(G)}(v_j)]$$

$$= \sum_{\{x,v_j\}\subset V(M(G))} \wedge [\mu(x,x_i) + \mu(x_i,v_j)][\sigma(x)[\sum_{i=1}^{n} \sigma(x_i)] + \sigma(v_j)[2d_G(v_j)]]$$

Since, $0 \le \sigma(x_i) \le 1$, $0 \le \mu(v_i, v_j) \le 1$, $\sum_{i=1}^{n} \sigma(x_i) \le n$ and $\delta \le d(u) \le \Delta$. Therefore,

$$DD^f(M(G)) \le \sum_{\{x,v_j\}\subset V(M(G))} 2[1(n) + 1(2\Delta)]$$

$$\le 2n(n + 2\Delta).$$

**Case 3.** If $u = v_i$ and $v = v_j$, such that $v_i$ and $v_j$ are adjacent vertices in $G$. Then the degree-distance index $DD^f(M(G))$ of fuzzy Mycielskian graph is given by:

$$DD^f(M(G)) = \sum_{\{v_i,v_j\}\subset V(M(G))} d_{M(G)}(v_i,v_j)[\sigma(v_i)d_{M(G)}(v_i) + \sigma(v_j)d_{M(G)}(v_j)].$$

By Observations 1 and 2 we have,

$$DD^f(M(G)) = \sum_{\{x,v_j\}\subset V(M(G))} [\mu(v_i,v_j)][\sigma(v_i)2d_G(v_i) + \sigma(v_j)2d_G(v_j)]$$

Since, $0 \le \sigma(x_i) \le 1$, $0 \le \mu(v_i,v_j) \le 1$ and $\delta \le d(u) \le \Delta$. Therefore,

$$DD^f(M(G)) \le \sum_{\{v_i,v_j\}\subset V(M(G))} 1[1(2\Delta) + 1(2\Delta)]$$

$$\le 4m\Delta.$$

**Case 4.** If $u = v_i$ and $v = v_j$, such that $v_i$ and $v_j$ are non-adjacent vertices in $G$. Then the degree-distance index $DD^f(M(G))$ of fuzzy Mycielskian graph is given by:

$$DD^f(M(G)) = \sum_{\{v_i,v_j\}\subset V(M(G))} d_{M(G)}(v_i,v_j)[\sigma(v_i)d_{M(G)}(v_i) + \sigma(v_j)d_{M(G)}(v_j)].$$

By Observations 1 and 2 we have,

$$DD^f(M(G)) = \sum_{\{x,v_j\}\subset V(M(G))} [\wedge(L_s(P_i: i = 1,2,3,\cdots))][\sigma(v_i)2d_G(v_i) + \sigma(v_j)2d_G(v_j)]$$

Since, $0 \le \sigma(x_i) \le 1$, $0 \le \mu(v_i,v_j) \le 1$ and $\delta \le d(u) \le \Delta$. Therefore,

$$DD^f(M(G)) \le \sum_{\{v_i,v_j\}\subset V(M(G))} [\wedge(L_s(P_i: i = 1,2,3,\cdots))][1(2\Delta) + 1(2\Delta)]$$

$$= 4\Delta \sum_{\{v_i,v_j\}\subset V(M(G))} [\wedge(L_s(P_i: i = 1,2,3,\cdots))]$$

$$= 4\Delta\chi(G),$$

where $\chi(G) = \sum_{\{v_i,v_j\}\subset V(M(G))} [\wedge(L_s(P_i: i = 1,2,3,\cdots))]$.

**Case 5.** If $u = v_i$ and $v = x_j$ where $x_j$ is the corresponding vertex of $v_j$ such that $v_i$ and $v_j$ are adjacent. Then the degree-distance index $DD^f(M(G))$ of fuzzy Mycielskian graph is given by:

$$DD^f(M(G)) = \sum_{\{v_i,x_j\}\subset V(M(G))} d_{M(G)}(v_i,x_j)[\sigma(v_i)d_{M(G)}(v_i) + \sigma(x_j)d_{M(G)}(x_j)].$$

By Observations 1 and 2 we have,

$$DD^f(M(G)) = \sum_{\{v_i,x_j\}\subset V(M(G))} [\mu(v_i,x_j)][\sigma(v_i)2d_G(v_i) + \sigma(x_j)[d_G(v_j) + \sigma(v_j)]]$$



Vol. 21, No. 1, (2024)
ISSN: 1005-0930

Since, $0 \leq \sigma(x_i) \leq 1$, $0 \leq \mu(v_i, v_j) \leq 1$ and $\delta \leq d(u) \leq \Delta$. Therefore,

$$DD^f(M(G)) \leq \sum_{\{v_i,x_j\}\subset V(M(G))} 1[1(2\Delta) + 1(\Delta + 1)]$$

$$\leq n(n-1)(3\Delta + 1).$$

**Case 6.** If $u = v_i$ and $v = x_i$ . Then the degree-distance index $DD^f(M(G))$ of fuzzy Mycielskian graph is given by:

$$DD^f(M(G)) = \sum_{\{v_i,x_i\}\subset V(M(G))} d_{M(G)}(v_i, x_i)[\sigma(v_i)d_{M(G)}(v_i) + \sigma(x_i)d_{M(G)}(x_i)].$$

By Observations 1 and 2 we have,

$$DD^f(M(G)) = \sum_{\{v_i,x_i\}\subset V(M(G))} \wedge [\mu(v_i, v_j) + \mu(v_j, x_i)][\sigma(v_i)2d_G(v_i) + \sigma(x_i)[d_G(v_i) + \sigma(v_i)]]$$

Since, $0 \leq \sigma(x_i) \leq 1$, $0 \leq \mu(v_i, v_j) \leq 1$ and $\delta \leq d(u) \leq \Delta$. Therefore,

$$DD^f(M(G)) \leq \sum_{\{v_i,x_i\}\subset V(M(G))} 2[1(2\Delta) + 1(\Delta + 1)]$$

$$\leq 2n(3\Delta + 1).$$

**Case 7.** If $u = v_i$ and $v = x_k$ where $x_k$ is the corresponding vertex of $v_k$ such that $v_i$ and $v_k$ are non-adjacent. Then the degree-distance index $DD^f(M(G))$ of fuzzy Mycielskian graph is given by:

$$DD^f(M(G)) = \sum_{\{v_i,x_k\}\subset V(M(G))} d_{M(G)}(v_i, x_k)[\sigma(v_i)d_{M(G)}(v_i) + \sigma(x_k)d_{M(G)}(x_k)].$$

By Observations 1 and 2 we have,

$$d_{M(G)}(v_i, x_k) \leq \wedge [\mu(v_i, x_j) + \mu(x_j, x) + \mu(x, x_k)]$$

or

$$d_{M(G)}(v_i, x_k) \leq \wedge [\mu(v_i, v_j) + \mu(v_j, x_k)]$$

Since, $0 \leq \mu(v_i, v_j) \leq 1$ . Therefore, $d_{M(G)}(v_i, x_k) \leq 3$ or $d_{M(G)}(v_i, x_k) \leq 2$ . Hence, we consider the maximum possibility in this case. Therefore,

$$DD^f(M(G)) \leq \sum_{\{v_i,x_k\}\subset V(M(G))} 3[\sigma(v_i)2d_G(v_i) + \sigma(x_k)[d_G(v_j) + \sigma(v_j)]]$$

Since, $0 \leq \sigma(x_i) \leq 1$ and $\delta \leq d(u) \leq \Delta$. Therefore,



Vol. 21, No. 1, (2024)
ISSN: 1005-0930

$$DD^f(M(G)) \leq \sum_{\{v_i, x_k\} \subset V(M(G))} 3[1(2\Delta) + 1(\Delta + 1)]$$
$$\leq 3n(n-1)(3\Delta + 1).$$

**Case 8.** If $u = x_i$ and $v = x_j$ . Then the degree-distance index $DD^f(M(G))$ of fuzzy Mycielskian graph is given by:

$$DD^f(M(G)) = \sum_{\{x_i, x_j\} \subset V(M(G))} d_{M(G)}(x_i, x_j)[\sigma(x_i) d_{M(G)}(x_i) + \sigma(x_j) d_{M(G)}(x_j)].$$

By Observations 1 and 2 we have,

$$d_{M(G)}(v_i, x_k) = [\mu(x_i, x) + \mu(x, x_j)]$$

or

$$d_{M(G)}(v_i, x_k) \leq \wedge [\mu(x_i, v_k) + \mu(v_k, x_j)]$$

Since, $0 \leq \mu(v_i, v_j) \leq 1$. Therefore, $d_{M(G)}(v_i, x_k) \leq 2$ Hence,

$$DD^f(M(G)) \leq \sum_{\{x_i, x_j\} \subset V(M(G))} 2[\sigma(x_i)[d_G(v_i) + \sigma(v_i)] + \sigma(x_j)[d_G(v_j) + \sigma(v_j)]]$$

Since, $0 \leq \sigma(x_i) \leq 1$ and $\delta \leq d(u) \leq \Delta$. Therefore,

$$DD^f(M(G)) \leq \sum_{\{x_i, x_j\} \subset V(M(G))} 2[1(\Delta + 1) + 1(\Delta + 1)]$$
$$\leq 2n(n-1)(\Delta + 1).$$

Combining the results from Cases (1)-(8), we get the desired result.

**Observation 4** *Let $G = (V, \sigma, \mu)$ be a fuzzy graph and $M^*(G)$ is partial complement of a Mycielskian fuzzy graph. Then*

- $deg_{M^*(G)}(v) = \sum_{i=1}^{n} \sigma(x_i)$ where $v = x$
- $deg_{M^*(G)}(v) = \sum_{v_i v_j \notin E(G)} \wedge [\sigma(v_i), \sigma(v_j)] + \sum_{v_i \in V(G), x_j \in X} \wedge [\sigma(v_i), \sigma(x_j)]$ where $v = v_i$, where $x_j$ is the corresponding vertex of $v_j$ such that $v_i$ and $v_j$ are non-adjacent
- $deg_{M^*(G)}(v) = \sum_{v_j \in V(G), x_i \in X} \wedge [\sigma(v_j), \sigma(x_i)] + \sigma(x_i)$ where $v = x_i$

**Observation 5** *Let $M^*(G)$ be a partial complement of a fuzzy Mycielskian graph $G = (V, \sigma, \mu)$. Then the distance between every pair of vertices in $M^*(G)$ are given below:*

- $d_{M^*(G)}(u,v) \leq \wedge [\mu(v_i,x_k)+\mu(x_k,x)+\mu(x,x_l)+\mu(x_l,v_j)]$ if $u=v_i$ and $v=v_j$ where $v_i$ and $v_j$ are adjacent in $G$ and there is no vertex $v_k \in V(G)$ such that $v_k$ is non-adjacent to both $v_i$ and $v_j$. or $\wedge [\mu(v_i,x_k)+\mu(x_k,v_j)]$ if there is a vertex $v_k \in V(G)$ such that $v_k$ is non-adjacent in $G$ to both $v_i$ and $v_j$.
- $d_{M^*(G)}(u,v) = \wedge [\sigma(v_i),\sigma(v_j)]$ if $u=v_i$ and $v=v_j$ where $v_i$ and $v_j$ are non-adjacent in $G$.
- $d_{M^*(G)}(u,v) = \wedge [\mu(v_i,v_j)+\mu(v_j,x_i)]$ if $u=v_i$ and $v=x_i$, where $v_i$ and $v_j$ are non-adjacent in $G$.
- $d_{M^*(G)}(u,v) = \wedge [\mu(v_i,x_k)+\mu(x_k,x)+\mu(x,x_j)]$ if $u=v_i$ and $v=x_j$ where $v_i$ and $v_j$ are adjacent in $G$ and there is no vertex $v_k \in V(G)$ such that $v_k$ is non-adjacent to both $v_i$ and $v_j$. or $\wedge [\mu(v_i,v_k)+\mu(v_k,x_j)]$ if there is a vertex $v_k \in V(G)$ such that $v_k$ is non-adjacent to both $v_i$ and $v_j$ in $G$.
- $d_{M^*(G)}(u,v) = [\mu(x_i,x)+\mu(x,x_j)]$ or $\wedge [\mu(x_i,v_k)+\mu(v_k,x_j)]$ if $u=x_i$ and $v=x_j$.
- $d_{M^*(G)}(u,v) = \wedge [\mu(v_i,x_k)+\mu(x_k,x)]$ if $u=v_i$ and $v=x$, where $v_i$ and $v_k$ are non-adjacent in $G$.
- $d_{M^*(G)}(u,v) = \sigma(x_i)$ if $u=x$ and $v=x_i$.

**Theorem 6** *Let $G=(V,\sigma,\mu)$ be a connected fuzzy graph with $n$-vertices and $m$-edges. Then the degree-distance index of $M^*(G)$ of a fuzzy graph $G$ is given by*

$$DD^f(M^*(G)) \leq 2[n(n-1)]^2 + n(n-1)[8m+2n+5] + n[3m+n+3].$$

*Proof.* Regarding to the different possible cases which $u$ and $v$ can be chosen from the set $V(M^*(G))$ the following cases are considered:

**Case 1.** If $u=x$ and $v=x_i$, then the length of the shortest $uv$-path is given by $\sigma(x_i)$.

Therefore, the degree-distance index $DD^f(M^*(G))$ is given by:

$$DD^f(M^*(G)) = \sum_{\{u,v\}\subseteq V(M^*(G))} d_{M^*(G)}(u,v)[\sigma(u)d_{M^*(G)}(u)+\sigma(v)d_{M^*(G)}(v)].$$

By Observations 4 and 5 we have,

$$DD^f(M^*(G)) = \sum_{\{x,x_i\}\subset V(M^*(G))} d_{M^*(G)}(x,x_i)[\sigma(x)d_{M^*(G)}(x)+\sigma(x_i)d_{M^*(G)}(x_i)]$$

$$= \sum_{\{x,x_i\}\subset V(M^*(G))} \sigma(x_i)[\sigma(x)[\sum_{i=1}^{n}\sigma(x_i)]$$

$$+\sigma(x_i)[\sum_{v_j\in V, x_i\in X} \wedge [\sigma(v_j),\sigma(x_i)]+\sigma(x_i)]]$$

Since, $0 \leq \sigma(u) \leq 1$, $0 \leq \mu(u,v) \leq 1$, $\sum_{i=1}^{n}\sigma(x_i) \leq n$. Therefore,



Vol. 21, No. 1, (2024)
ISSN: 1005-0930

$$DD^f(M^*(G)) \leq \sum_{\{x,x_i\}\subset V(M^*(G))} 1[1(n) + 1(m+1)]$$

$$\leq n(n+m+1).$$

**Case 2.** If $u = x$ and $v = v_i$, then the degree-distance index $DD^f(M^*(G))$ is given by:

$$DD^f(M^*(G)) = \sum_{\{x,v_i\}\subset V(M^*(G))} d_{M^*(G)}(x, v_i)[\sigma(x)d_{M^*(G)}(x) + \sigma(v_i)d_{M^*(G)}(v_i)].$$

By Observations 4 and 5 we have, $d_{M^*(G)}(u, v) = \wedge[\mu(v_i, x_k) + \mu(x_k, x)]$ if $u = v_i$ and $v = x$, where $v_i$ and $v_k$ are non-adjacent in $G$. Hence

$$DD^f(M^*(G)) = \sum_{\{x,v_j\}\subset V(M^*(G))} \wedge[\mu(v_i, x_k) + \mu(x_k, x)][\sigma(x)d_{M^*(G)}(x) + \sigma(v_i)d_{M^*(G)}(v_i)]$$

$$= \sum_{\{x,v_i\}\subset V(M^*(G))} \wedge[\mu(v_i, x_k) + \mu(x_k, x)]$$

$$[\sigma(x)[\sum_{i=1}^{n} \sigma(x_i)] + \sigma(v_i)[\sum_{v_iv_j\notin E(G)} \wedge[\sigma(v_i), \sigma(v_j)]$$

$$+ \sum_{v_i\in V(G), x_j\in X} \wedge[\sigma(v_i), \sigma(x_j)]]]$$

Since, $0 \leq \sigma(u) \leq 1$, $0 \leq \mu(u, v) \leq 1$, $\sum_{i=1}^{n} \sigma(x_i) \leq n$. Therefore,

$$DD^f(M^*(G)) \leq \sum_{\{x,v_i\}\subset V(M^*(G))} 2[1(\frac{n(n-1)}{2} - m) + 1(m)]$$

$$\leq n^2(n-1).$$

**Case 3.** If $u = v_i$ and $v = v_j$, such that $v_i$ and $v_j$ are adjacent vertices in $G$ and there is no vertex $v_k \in V(G)$ such that $v_k$ is non-adjacent to both $v_i$ and $v_j$.. Then the degree-distance index $DD^f(M^*(G))$ is given by:

$$DD^f(M^*(G)) = \sum_{\{v_i,v_j\}\subset V(M^*(G))} d_{M^*(G)}(v_i, v_j)[\sigma(v_i)d_{M^*(G)}(v_i) + \sigma(v_j)d_{M^*(G)}(v_j)].$$

By Observations 4 and 5 we have,

$$d_{M^*(G)}(u, v) \leq \wedge[\mu(v_i, x_k) + \mu(x_k, x) + \mu(x, x_l) + \mu(x_l, v_j)]$$

or

$$d_{M^*(G)}(u, v) \leq \wedge[\mu(v_i, x_k) + \mu(x_k, v_j)]$$



Vol. 21, No. 1, (2024)
ISSN: 1005-0930

Since $0 \leq \mu(u,v) \leq 1$, therefore, $d_{M^*(G)}(u,v) \leq 4$ or $d_{M^*(G)}(u,v) \leq 2$. Here we consider the maximum possibility of distance between $v_i$ and $v_i$. Hence

$$DD^f(M^*(G)) \leq \sum_{\{v_i,v_j\}\subset V(M^*(G))} 4[\sigma(v_i)[\sum_{v_iv_j\notin E(G)} \wedge [\sigma(v_i),\sigma(v_j)] + \sum_{v_i\in V(G),x_j\in X} \wedge [\sigma(v_i),\sigma(x_j)]]$$

$$+\sigma(v_j)[\sum_{v_iv_j\notin E(G)} \wedge [\sigma(v_j),\sigma(v_i)] + \sum_{v_j\in V(G),x_i\in X} \wedge [\sigma(v_j),\sigma(x_i)]]]$$

Since, $0 \leq \sigma(u) \leq 1$. Therefore,

$$DD^f(M^*(G)) \leq \sum_{\{v_i,v_j\}\subset V(M^*(G))} 4[\frac{n(n-1)}{2} - m + m + \frac{n(n-1)}{2} - m + m$$

$$\leq 4mn(n-1).$$

**Case 4.** If $u = v_i$ and $v = v_j$, such that $v_i$ and $v_j$ are non-adjacent vertices in $G$. Then the degree-distance index $DD^f(M^*(G))$ is given by:

$$DD^f(M^*(G)) = \sum_{\{v_i,v_j\}\subset V(M^*(G))} d_{M^*(G)}(v_i,v_j)[\sigma(v_i)d_{M^*(G)}(v_i) + \sigma(v_j)d_{M^*(G)}(v_j)].$$

By Observations 4 and 5 we have,

$$DD^f(M^*(G)) \leq \sum_{\{v_i,v_j\}\subset V(M^*(G))} \wedge [\sigma(v_i),\sigma(v_j)]$$

$$[\sigma(v_i)[\sum_{v_iv_j\notin E(G)} \wedge [\sigma(v_i),\sigma(v_j)] + \sum_{v_i\in V(G),x_j\in X} \wedge [\sigma(v_i),\sigma(x_j)]]$$

$$+\sigma(v_j)[\sum_{v_iv_j\notin E(G)} \wedge [\sigma(v_j),\sigma(v_i)] + \sum_{v_j\in V(G),x_i\in X} \wedge [\sigma(v_j),\sigma(x_i)]]]$$

Since, $0 \leq \sigma(u) \leq 1$ and $0 \leq \mu(u,v) \leq 1$. Therefore,

$$DD^f(M^*(G)) \leq \sum_{\{v_i,v_j\}\subset V(M^*(G))} [\frac{n(n-1)}{2} - m + m + \frac{n(n-1)}{2} - m + m]$$

$$\leq (\frac{n(n-1)}{2} - m)(n(n-1))$$

$$= \frac{[n(n-1)]^2}{2} - mn(n-1).$$



Vol. 21, No. 1, (2024)
ISSN: 1005-0930

**Case 5.** If $u = v_i$ and $v = x_j$ where $v_i$ and $v_j$ are adjacent and there is no vertex $v_k \in V(G)$ such that $v_k$ is non-adjacent to both $v_i$ and $v_j$. Then the degree-distance index $DD^f(M^*(G))$ is given by:

$$DD^f(M^*(G)) = \sum_{\{v_i,x_j\}\subset V(M^*(G))} d_{M^*(G)}(v_i,x_j)[\sigma(v_i)d_{M^*(G)}(v_i) + \sigma(x_j)d_{M^*(G)}(x_j)].$$

By Observations 4 and 5 we have, $d_{M^*(G)}(v_i,x_j) = \wedge [\mu(v_i,x_k) + \mu(x_k,x) + \mu(x,x_j)]$ or $d_{M^*(G)}(v_i,x_j) = \wedge [\mu(v_i,v_k) + \mu(v_k,x_j)]$. Since, $0 \leq \mu(u,v) \leq 1$. Therefore, $d_{M^*(G)}(v_i,x_j) = 3$ or $d_{M^*(G)}(v_i,x_j) = 2$. Here we consider the maximum possibility of distance between $v_i$ and $x_j$.

$$DD^f(M^*(G)) = \sum_{\{v_i,x_j\}\subset V(M^*(G))} 3[\sigma(v_i)[\sum_{v_iv_j\notin E(G)} \wedge [\sigma(v_i),\sigma(v_j)] + \sum_{v_i\in V(G),x_j\in X} \wedge [\sigma(v_i),\sigma(x_j)]] + \sigma(x_j)[\sum_{v_i\in V,x_j\in X} \wedge [\sigma(v_i),\sigma(x_j)] + \sigma(x_j)]]$$

Since, $0 \leq \sigma(u) \leq 1$. Therefore,

$$DD^f(M^*(G)) \leq \sum_{\{v_i,x_j\}\subset V(M^*(G))} 3[\frac{n(n-1)}{2} + m + 1]$$

$$= 3n(n-1)[\frac{n(n-1)}{2} + m + 1]$$

$$= \frac{3[n(n-1)]^2}{2} + 3mn(n-1) + 3n(n-1).$$

**Case 6.** If $u = v_i$ and $v = x_i$. Then the degree-distance index $DD^f(M^*(G))$ is given by:

$$DD^f(M^*(G)) = \sum_{\{v_i,x_i\}\subset V(M^*(G))} d_{M^*(G)}(v_i,x_i)[\sigma(v_i)d_{M^*(G)}(v_i) + \sigma(x_i)d_{M^*(G)}(x_i)].$$

By Observations 4 and 5 we have,

$$DD^f(M^*(G)) = \sum_{\{v_i,x_i\}\subset V(M^*(G))} \wedge [\mu(v_i,v_j) + \mu(v_j,x_i)]$$

$$[\sigma(v_i)[\sum_{v_iv_j\notin E(G)} \wedge [\sigma(v_i),\sigma(v_j)] + \sum_{v_i\in V(G),x_i\in X} \wedge [\sigma(v_i),\sigma(x_i)]]$$

$$+\sigma(x_i)[\sum_{v_j \in V, x_i \in X} \wedge [\sigma(v_j), \sigma(x_i)] + \sigma(x_i)]]$$

Since, $0 \leq \sigma(u) \leq 1$ and $0 \leq \mu(u,v) \leq 1$. Therefore,

$$DD^f(M^*(G)) \leq \sum_{\{v_i, x_i\} \subset V(M^*(G))} 2[\frac{n(n-1)}{2} + m + 1]$$

$$= 2n[\frac{n(n-1)}{2} + m + 1]$$

$$= n^2(n-1) + 2mn + 2n.$$

**Case 7.** If $u = x_i$ and $v = x_j$. Then the degree-distance index $DD^f(M^*(G))$ is given by:

$$DD^f(M^*(G)) = \sum_{\{x_i, x_j\} \subset V(M^*(G))} d_{M^*(G)}(x_i, x_j)[\sigma(x_i) d_{M^*(G)}(x_i) + \sigma(x_j) d_{M^*(G)}(x_j)].$$

By Observations 4 and 5 we have,

$d_{M^*(G)}(x_i, x_j) = [\mu(x_i, x) + \mu(x, x_j)$ or $d_{M^*(G)}(x_i, x_j) = \wedge [\mu(x_i, v_k) + \mu(v_k, x_j)]$ . Since $0 \leq \mu(u,v) \leq 1$. Therefore, $d_{M^*(G)}(x_i, x_j) = 2$. Hence

$$DD^f(M^*(G)) \leq \sum_{\{x_i, x_j\} \subset V(M^*(G))} 2[\sigma(x_i)[\sum_{v_j \in V, x_i \in X} \wedge [\sigma(v_j), \sigma(x_i)] + \sigma(x_i)]$$

$$+\sigma(x_j)[\sum_{v_i \in V, x_j \in X} \wedge [\sigma(v_i), \sigma(x_j)] + \sigma(x_j)]]$$

Since, $0 \leq \sigma(u) \leq 1$ and $0 \leq \mu(u,v) \leq 1$. Therefore,

$$DD^f(M^*(G)) \leq \sum_{\{x_i, x_j\} \subset V(M^*(G))} 2[m + 1 + m + 1]$$

$$= \frac{n(n-1)}{2} \times 2 \times 2(m+1)$$

$$= 2n(m+1)(n-1).$$

Combining the results from Cases (1)-(7), we get the desired result.

## 3 Conclusion

In this paper the degree-distance index of fuzzy graphs has been put forwarded and studied this concept for fuzzy Mycielskian graph and its partial complement. Further results on this parameter is open for the researchers.

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