JOURNAL OF BASIC SCIENCE AND ENGINEERING

FUZZY MINIMAL G#rg-OPEN SETS AND FUZZY MAXIMAL G#rg-OPEN SETS IN FUZZY TOPOLOGICAL SPACES

***Holabasayya I. Sankannavar, Jenifer J. Karnel**

Department of Mathematics

KLE's Polytechnic, Bailhongal-591102, Karnataka-India

SDM College of Engineering and Technology, Dharwad-580002, Karnataka-India

DOI: https://doie.org/10.0628/Jbse.2024959424

Abstract: The aim of this paper is to introduce fuzzy minimal G#rg-open and fuzzy maximal G#rg-open sets in fuzzy topological space. Further, we investigate related properties with these new sets.

2010 AMS Classification: 54A40

Keywords and phrases: fuzzy minimal open sets, fuzzy G#rg-open sets, fuzzy minimal G#rg-open sets, fuzzy maximal G#rg-open sets.

1. Introduction: In 1965 Zadeh [5] established concept of fuzzy set and in 1968 Chang [2] introduced fuzzy topology and in 1981 Azad[1] investigated fuzzy semi-continuity properties in fts. Ittanagi and Wali [3] instigated the notions of fuzzy maximal and minimal open sets. Recently the notion of fuzzy *G#rg*-closed set introduced and investigated by Holabasayya Sankannavar and Jenifer Karnel[4]. This paper, introduce new class of fuzzy minimal G#rg-open and fuzzy maximal G#rg-open sets. Further some of their related properties investigated.

Throughout this paper *fts* refers to *fuzzy topological space*.

Definition 1.1: A proper nonempty fuzzy open subset U of a fts X is said to be fuzzy minimal open set, if any fuzzy open set which is contained in U is 0_X or U.

Definition 1.2: A proper nonempty fuzzy open subset U of a fts X is said to be fuzzy maximal open set, if any fuzzy open set which contains U is 1_X or U.

Definition 1.3: A proper nonempty fuzzy closed subset F of afts X is said to be fuzzy minimal closed set, if any fuzzy closed set which is contained in F is 0_X or F.

Definition 1.4: A proper nonempty fuzzy closed subset F of a fts X is said to be fuzzy maximal closed set, if any fuzzy closed set which contains F is 1_X or F.

Definition 1.5:[4] A fuzzy subset A of a fts X is said to be fuzzy generalized #rg-closed (briefly, Fg#rg-closed) set, if cl(A) ≤ U, whenever A ≤ U and U is fuzzy #rg-open set in X. The complement of fuzzy G#rg-closed set is their fuzzy G#rg-open set in fts X.

Definition 1.6: Let A be fuzzy subset of fts X, then fuzzy G#rg-closure of A is defined as Fg#rg-cl(A) = ⋀{ all fuzzy G#rg-closed sets containing the fuzzy set A}.

2.Minimal G#rg-open sets and maximal G#rg-closed sets:

A new class of sets called fuzzy minimal G#rg-open sets (fuzzy maximal G#rg-closed sets) and fuzzy maximal G#rg-open sets (fuzzy minimal G#rg-closed sets)in fuzzy topological spaces are introduced, which are subclasses of Fg#rg-open sets (Fg#rg-closed sets). We investigate some of their properties infuzzy topological spaces.

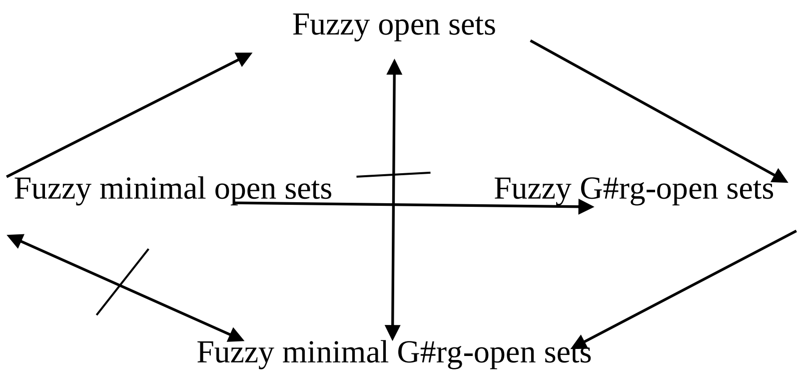
Definition 2.1:Let U be any fuzzy G#rg-open subset of ftsX, U is called fuzzy minimal G#rg-open set if and only if any fuzzy G#rg-open set which is contained in set U is 0_X or U.

Remark 2.2:Fuzzy minimal open sets and fuzzy minimal G#rg-open sets are independent andfuzzy open sets and fuzzy minimal G#rg-open sets are independent each other and is illustrated in the following example.

Example 2.3**:**Let X = {a, b, c, d} be any fuzzy set and fuzzy subsets are 0_x ={(a, 0), (b, 0), (c, 0), (d, 0)} = 0, α_1 = {(a, 1), (b, 0), (c, 0), (d, 0)}, α_2 ={(a, 0), (b, 1), (c, 1), (d, 0)}, α_3 ={(a, 1), (b, 1), (c, 1), (d, 0)} and 1_x ={(a, 1), (b, 1), (c, 1), (d, 1)}= 1. The fuzzy topology of X is T = {0_x, α_1, α_2, α_3, 1_x}, then fuzzy minimal open sets are α_1 = {(a, 1), (b, 0), (c, 0), (d, 0)} and α_2 ={(a, 0), (b, 1), (c, 1), (d, 0)}. Let fuzzy G#rg-open sets in fts X are 1_X, 0_X, α_1 = {(a, 1), (b, 0), (c, 0), (d, 0)}, β_1 = {(a, 0), (b, 1), (c, 0), (d, 0)}, β_2 = {(a, 0), (b, 0), (c, 1), (d, 0)}, β_3 = {(a, 1), (b, 1), (c, 0), (d, 0)}, β_4 = {(a, 1), (b, 0), (c, 1), (d, 0)}, α_3 = {(a, 1), (b, 1), (c, 1), (d, 0)} and fuzzy minimal G#rg-open sets are α_1, β_1,β_2. Here α_3 is a fuzzy minimal open set (fuzzy open set) but is not a fuzzy G#rg-open set and the fuzzy sets α_1 and β_1 are fuzzy minimal G#rg-open sets, but fuzzy minimal open sets (fuzzy sets).

Remark 2.4**:** From above discussion we have the following implications:

Vol. 21, No. 1, (2024)
ISSN: 1005-0930



Where, A → B means A implies B and A ↮ B means A is independent with B.

Theorem 2.5: i) Let G be fuzzy minimal G#rg-open set and H be fuzzy G#rg-open set, then $G \wedge H = 0_X$ or $G \leq H$.

ii) Let G and H be fuzzy minimal G#rg-open sets, then $G \wedge H = 0_X$ or G = H.

*Proof***:** (i) Let G be fuzzy minimal G#rg-open set and H be fuzzy G#rg-open set. If$G \wedge H = 0_X$, then there is nothing to prove.But if $G \wedge H \neq 0_X$, then we have to prove that $G \leq H$. Suppose $G \wedge H \neq 0_X$. Then $G \wedge H \leq U$ and $G \wedge H$ is fuzzy G#rg-open set, as the finite intersection of fuzzy G#rg-open sets is a fuzzy G#rg-open set. Since G is a fuzzy minimal G#rg-open set, we have $G \wedge H = G$. Therefore $G \leq H$.

(ii) Let G and H are fuzzy minimal G#rg-open sets. Suppose,$G \wedge H \neq 0_X$, then we see that $G \leq H$ and $H \leq G$ by (i). Therefore G = H.

Theorem 2.6**:** Let G be a fuzzy minimal G#rg-open set. If x_α is an element of G, then $G \leq H$ for any fuzzy open neighbourhood H of x_α.

*Proof***:** Let G be fuzzy minimal G#rg-open set and x_α be an element of G. Suppose there existsfuzzy open neighbourhood H of x_α such that $G \nleq H$. Then $G \wedge H$ is a fuzzy G#rg-open set such that $G \wedge H \leq G$ and $G \wedge H \neq 0_X$. Since G is a fuzzy minimal G#rg-open set, we have $G \wedge H = G$ i.e. $G \leq H$. This contradicts our assumption that $G \nleq H$. Therefore, $G \leq H$ for any fuzzy open neighbourhood H of x_α.

Theorem 2.7**:**Let G be fuzzy minimal G#rg-open set. If x_α is an element of G, then $G \leq H$ for any fuzzy G#rg-open set H containing x_α.

Vol. 21, No. 1, (2024)
ISSN: 1005-0930

*Proof***:** Let G be fuzzy minimal G#rg-open set containing an elementx_α. Suppose there exists a fuzzyG#rg-open set H containing x_α such that G $\nleq$ H. Then G ∧ H is a fuzzy G#rg-open set such that G ∧H ≤ G and G ∧ H ≠ 0_X. Since G is a fuzzy minimal G#rg-open set, we have G ∧H = G i.e. G ≤H. This contradicts our assumption that G $\nleq$ H. Therefore G ≤ H for any fuzzy G#rg-open set H containing x_α.

Theorem 2.8: Let G be fuzzy minimal G#rg-open set,then G =∧{H: H is any fuzzy G#rg-open set containing x_α} for any element x_α of G.

*Proof***:** By Theorem* and from the fact that G is fuzzy G#rg-open set containing x_α, we have G ≤ ∧ {H: H is any fuzzy G#rg-open set containing x_α}≤ G. Therefore, we have the result.

Theorem 2.9**:** Let G be a nonempty G#rg-open set, then the following three conditions are equivalent.

i) G is fuzzy minimal G#rg-open set.
ii) G ≤ Fg#rg-cl(A) for any nonempty fuzzy subset A of G.
iii) Fg#rg-cl(G)=Fg#rg-cl(A) for any nonempty fuzzy subset A of G.

*Proof***:** (1)⇒ (2) Let G be a fuzzy minimal G#rg-open set, x_α ∈G and A be a nonempty fuzzy subset of G. By Theorem*, for any fuzzy G#rg-open set H containing x_α, A ≤G ≤H which implies A ≤ H. Now A = A∧G ≤ A ∧ H. Since A is nonempty, therefore A ∧ H≠ 0_X. Since H is any fuzzy G#rg-open set containing x_α, by the property, x_α ∈ Fg#rg-cl(A). That is x_α ∈ G implies x_α ∈Fg#rg-cl(A) which implies G≤ Fg#rg-cl(A) for any nonempty fuzzy subset A of G.

(2) ⇒ (3) Let A be fuzzy nonempty subset of G. That is A≤G which implies Fg#rg-cl(A) ≤ Fg#rg-cl(G)---(i). Again from (2) G≤ Fg#rg-cl(A) for any non-empty fuzzy subset A of G which implies Fg#rg-cl(G) ≤ Fg#rg-cl(Fg#rg-cl(A))= Fg#rg-cl(A). That is Fg#rg-cl(G)≤ Fg#rg-cl(A)--- (ii). From (i) and (ii), we have Fg#rg-cl(G)= Fg#rg-cl(A) for any nonempty fuzzy subset A of G.

(3) ⇒ (1) From (3) we have Fg#rg-cl(G) = Fg#rg-cl(A) for any nonempty fuzzy subsetA of G. Suppose G is not a fuzzy minimal G#rg-open set. Then there exists a nonempty fuzzy G#rg-open set I such that I ≤G and I≠G. Now there exists an element (a,1)∈G such that (a,1)∉ I which implies(a,1)∈ 1_X−I. That is Fg#rg-cl({(a,1)})≤ Fg#rg-cl(1_X−I)= 1_X−I, as 1_X−I is a fuzzy G#rg-closed set in X. It follows that Fg#rg-cl({(a,1)})≠ Fg#rg-cl(G). This is

Vol. 21, No. 1, (2024)
ISSN: 1005-0930

a contradiction to fact that Fg#rg-cl({(a,1)})=Fg#rg-cl(G) for any nonempty fuzzy subset {(a,1)} of G. Therefore, G is a fuzzy minimal G#rg-open set.

Theorem 2.10: Let G be fuzzy nonempty finite fuzzy G#rg-open set, then there exists at least one (finite) fuzzy minimal G#rg-open set H such that H≤G.

Proof: Let G be nonempty finite fuzzy G#rg-open set. If G is a fuzzy minimal G#rg-open set, we may set H = G. If G is not a fuzzy minimal G#rg-open set, then there exists a (finite) fuzzyG#rg-open set G_1 such that $0_X \neq G_1 \leq G$. If G_1 is a fuzzy minimal G#rg-open set, we may set H = G_1 If G_1 is not a fuzzy minimal G#rg-open set, then there exists a (finite) fuzzyG#rg-open set G_2 such that $0_X \neq G_2 \leq G_1$. Continuing this process, we have a sequence of fuzzy G#rg-open sets $G > G_1 > G_2 > G_3 \ldots > G_k > \ldots$. Since G is a finite fuzzy set, this process repeats only finitely. Then finally we get a fuzzy minimal G#rg-open set H =G_n for some positive integer n.

Corollary 2.11**:** Let G be a finite fuzzy minimal open set,then there exists at least one (finite) fuzzy minimal G#rg-open set H such that H≤G.

*Proof***:** Let G be a fuzzy finite minimal open set, then G is a nonempty finite fuzzy G#rg-open set. By theorem 2.10, there exists at least one (finite) fuzzy minimal G#rg-open set H such that H≤G.

Theorem 2.12**:** Let G and G_λ arefuzzy minimal G#rg-open sets for any element λ of Λ. If $G \leq \bigvee_{\lambda\in\Lambda} G_\lambda$, then there exists an element λ of Λ such that G = G_λ.

Proof: Let $G \leq \bigvee_{\lambda\in\Lambda} G_\lambda$. Then $G \wedge (\bigvee_{\lambda\in\Lambda} G_\lambda)$= G. That is$\bigvee_{\lambda\in\Lambda}(G \wedge G_\lambda)$ = G. Also, by theorem 5(ii), $\bigvee_{\lambda\in\Lambda}(G \wedge G_\lambda)$= 0_X or G =G_λ for any λ ∈ Λ. It follows that there exists an element λ ∈ Λ such that G =G_λ.

Theorem 2.13**:** Let G and $G_\lambda$are fuzzy minimal G#rg-open sets for any element λ of Λ. If G =$G_\lambda$for any element$\lambda \in \Lambda$, then$(\bigvee_{\lambda\in\Lambda} G_\lambda) \wedge G = 0_X$.

Proof: Suppose that $(\bigvee_{\lambda\in\Lambda} G_\lambda) \wedge G \neq 0_X$. That is $\bigvee_{\lambda\in\Lambda}(G_\lambda \wedge G) \neq 0_X$. Then there exits an element $\lambda \in \Lambda$ such that$G_\lambda \wedge G \neq 0_X$. By theorem 5(ii), we have G=G_λ, which contradicts the fact that $G \neq G_\lambda$ for any $\lambda \in \Lambda$. Hence$(\bigvee_{\lambda\in\Lambda} G_\lambda) \wedge G = 0_X$.

Vol. 21, No. 1, (2024)
ISSN: 1005-0930

Theorem 2.14**:** Let $G_\lambda$be fuzzy minimal G#rg-open set for any element λ of Λ and $G_\lambda \wedge G_\mu$ for any elements λ and μ of Λ with $\lambda \neq \mu$.Assume that $|\Lambda| \geq 2$. Let μ be any element of Λ, then $(\vee_{\lambda\in\Lambda-(M)} G_\lambda) \wedge G_\mu = 0_X$.

*Proof***:** Put G by$G_\lambda$in theorem 2.12, then we have the result.

Corollary 2.15**:** Let G_λ be a fuzzy minimal G#rg-open set for any element λ of Λ and $G_\lambda \neq G_\mu$ forany elements λ and μ of Λ with $\lambda \neq \mu$. If T is a proper nonempty fuzzy subset of Λ, then $(\vee_{\lambda\in\Lambda-T} G_\lambda) \wedge (\vee_{\gamma\in T} G_\gamma) = 0_X$.

Theorem 2.16**:** Let G_λ and G_γ are fuzzy minimal G#rg-open sets for any element $\lambda \in \Lambda$and $\gamma \in T$. If there exists an element $\gamma$of T such that $G_\lambda \neq G_\gamma$for any element λ of Λ, then $(\vee_{\gamma\in T} G_\gamma) \nless (\vee_{\lambda\in\Lambda} G_\lambda)$.

*Proof***:** Suppose that an element γ^1 of T satisfies $G_\lambda \neq G_{\gamma 1}$ for any element γ of Λ. If $(\vee_{\gamma\in T} G_\gamma) < (\vee_{\lambda\in\Lambda} G_\lambda)$, then we see $G_{\gamma 1} < \vee_{\lambda\in\Lambda} G_\lambda$. By theorem 2.12, there exists an element λ of Λ such that $G_{\gamma 1} = G_\lambda$, which is a contradiction. It follows that $(\vee_{\gamma\in T} G_\gamma) \nless (\vee_{\lambda\in\Lambda} G_\lambda)$.

Theorem 2.17**:** Let G_λ be a fuzzy minimal G#rg-open set for any element λ of Λ and $G_\lambda \neq G_\mu$ for any elements λ and μ of Λ with $\lambda \neq \mu$. If T is a proper nonempty fuzzy subset of Λ, then $(\vee_{\gamma\in T} G_\gamma) < (\vee_{\lambda\in\Lambda} G_\lambda)$.

*Proof***:** Let k be any element of Λ-T, then $G_k \wedge \vee_{\gamma\in T} G_\gamma = \vee_{\gamma\in T}(G_k \wedge G_\gamma) = 0_X$ and $G_k \wedge (\vee_{\lambda\in\Lambda} G_\lambda) = \vee_{\lambda\in\Lambda}(G_k \wedge G_\lambda) = G_k$.If $(\vee_{\gamma\in T} G_\gamma) = (\vee_{\lambda\in\Lambda} G_\lambda)$, then we have $0_X = G_k$. This contradicts our assumption that G_k is a fuzzy minimal G#rg-open set. Therefore, we have the result.

Definition: Let F be any proper fuzzyG#rg-closed subset of X is called fuzzy maximal G#rg-closed set if and only if any fuzzyG#rg-closed set which contains F is either 1_X or F.

Remark 2.18: Fuzzy Maximal closed sets and fuzzy maximal G#rg-closed sets are independent each other as illustrated as,

Example 2.19:Let X = {a, b, c, d} be any fuzzy set and fuzzy subsets are 0_x ={(a, 0), (b, 0), (c, 0), (d, 0)} = 0, α_1 = {(a, 1), (b, 0), (c, 0), (d, 0)}, α_2 ={(a, 0), (b, 1), (c, 0), (d, 0)}, α_3 ={(a, 0), (b, 0), (c, 1), (d, 0)}, α_4 ={(a, 0), (b, 0), (c, 0), (d, 1)}, α_5 = {(a, 1), (b, 1), (c, 0), (d, 0)}, α_6 = {(a, 1), (b, 0), (c, 1), (d, 0)}, α_7 = {(a, 1), (b, 0), (c, 0), (d, 1)}, α_8 = {(a, 0), (b, 1), (c, 1),

Vol. 21, No. 1, (2024)
ISSN: 1005-0930

(d, 0)}, α_9 = {(a, 0), (b, 1), (c, 0), (d, 1)}, α_{10} = {(a, 0), (b, 0), (c, 1), (d, 1)}, α_{11} = {(a, 1), (b, 1), (c, 1), (d, 0)}, α_{12} = {(a, 1), (b, 1), (c, 0), (d, 1)}, α_{13} = {(a, 1), (b, 0), (c, 1), (d, 1)}, α_{14} = {(a, 0), (b, 1), (c, 1), (d, 1)} and 1_x ={(a, 1), (b, 1), (c, 1), (d, 1)}= 1, with the fuzzy topology of X is T = {0_x, α_1, α_8, α_{11}, 1_x}, then fuzzy closed sets in X are0_x, α_4, α_7, α_{14}, 1_x. The fuzzy maximal closed sets are α_7, α_{14} and fuzzy G#rg-closed sets are0_X, α_4, α_7, α_9, α_{10}, α_{12}, α_{13}, α_{14}, 1_x,then fuzzy maximal G#rg-closed sets are α_{12}, α_{13}, α_{14}. But$\alpha_7$is fuzzy maximal closed set but not fuzzy maximal G#rg-closed set and also α_{13} is fuzzy maximal G#rg-closed sets but not fuzzy maximal closed set in fts X.

Theorem 2.20: Let F be any proper fuzzy subset of X is said to be fuzzy maximal closed set iff $1_X - F$ is fuzzy minimal G#rg-open set in X.

*Proof***:** Let F be any fuzzy maximal G#rg-closed set. Suppose $1_X - F$ is not a fuzzy minimal G#rg-open set, then there exists fuzzy G#rg-open set $U \neq 1_X - F$ such that $0_X \neq U < 1_X - F$. That is $F < 1_X - U$ and $1_X - U$ is a fuzzy G#rg-closed set. This contradicts our assumption that F is a fuzzy minimal G#rg-open set.

Conversely, let $1_X - F$ be a fuzzy minimal G#rg-open set. Suppose F is not a fuzzy maximal G#rg-closed set, then there exists a fuzzy G#rg-closed set $E \neq F$ such that $F < E \neq 1_X$. That is $0_X \neq 1_X - E < 1_X - F$ and $1_X - E$ is a fuzzy G#rg-open set. This contradicts our assumption that $1_X - F$ is a fuzzy minimalG#rg-open set. Therefore, F is a fuzzy maximal G#rg-closed set.

Theorem 2.21**:** i) Let F be a fuzzy maximal G#rg-closed set and E be any fuzzy G#rg-closed set, then $F \vee E = 1_X$ or $E < F$.

ii) Let F and E arefuzzy maximal G#rg-closed sets, then $F \vee E = 1_X$ or F = E.

*Proof***:** i) Let F be fuzzy maximal G#rg-closed set and E be any fuzzy G#rg-closed set. If $F \vee E = 1_X$, then there is nothing to prove. But if $F \vee E \neq 1_X$, then we have to prove that $E < F$. Suppose $F \vee E \neq 1_X$,then $F < F \vee E$ and $F \vee E$ is fuzzy G#rg-closed set, as the finite fuzzy union of fuzzy G#rg-closed sets is a fuzzy G#rg-closed set, we have $F \vee E = 1_X$ or $F \vee E = F$. Therefore, $F \vee E = F$ which implies $E < F$.

ii) Let F and E are fuzzy maximal G#rg-closed sets. Suppose $F \vee E \neq 1_X$, then we see that $F < E$ and $E < F$ by (i). Therefore F = E.

Vol. 21, No. 1, (2024)
ISSN: 1005-0930

Theorem 2.22**:** Let F be a fuzzy maximal G#rg-closed set. If x_α is a fuzzy element of F, then for any fuzzy G#rg-closed set E containing x_α, F ⋁ E = 1_X or E< F.

*Proof***:** Let F be a fuzzy maximal G#rg-closed set and x_α is a fuzzy element of F. Suppose there exists a fuzzy G#rg-closed set E containing x_α such that F ⋁ E≠ X. Then F < F ⋁E and F ⋁ E is a G#rg-closed set, as the finite union of G#rg-closed sets is a G#rg-closed set. Since F is a G#rg-closed set, we have F ⋁ E = F. Therefore E< F.

Theorem 2.23: LetF_α,F_β,$F_\gamma$are fuzzy maximal G#rg-closed sets such that$F_\alpha \neq F_\beta$. If $F_\alpha \wedge F_\beta < F_\gamma$, then either $F_\alpha = F_\gamma$ or $F_\beta = F_\gamma$.

Proof: Given that $F_\alpha \wedge F_\beta < F_\gamma$. If $F_\alpha = F_\gamma$ then there is nothing to prove. But if $F_\alpha \neq F_\gamma$ then we must prove $F_\beta = F_\gamma$. Now $F_\beta \wedge F_\gamma = F_\beta \wedge (F_\gamma \wedge 1_X) = F_\beta \wedge (F_\gamma \wedge (F_\alpha \vee F_\beta)$ (by theorem 2.21(ii) = $F_\beta \wedge ((F_\gamma \wedge F_\alpha) \vee (F_\gamma \wedge F_\beta))$= $(F_\beta \wedge F_\gamma \wedge F_\alpha) \vee (F_\beta \wedge F_\gamma \wedge F_\beta)$= $(F_\alpha \wedge F_\beta) \vee (F_\gamma \wedge F_\beta)$ (by $F_\alpha \wedge F_\beta < F_\gamma$)= $(F_\alpha \vee F_\gamma) \wedge F_\beta$= $1_X \wedge F_\beta$ (Since F_α and F_γ are fuzzy maximal G#rg-closed sets by theorem 2.21(ii), $F_\alpha \vee F_\gamma = 1_X$) = F_β. That is $F_\beta \wedge F_\gamma = F_\beta$ which implies $F_\beta < F_\gamma$. SinceF_β and F_γ are fuzzy maximal G#rg-closed sets, we have $F_\beta = F_\gamma$. Therefore $F_\beta = F_\gamma$.

Theorem 2.24: LetF_α,F_β and F_γ be fuzzy maximal G#rg-closed sets which are different from each other. Then $(F_\alpha \wedge F_\beta) \nless (F_\alpha \wedge F_\gamma)$.

Proof: Let $(F_\alpha \wedge F_\beta) < (F_\alpha \wedge F_\gamma)$ which implies $(F_\alpha \wedge F_\beta) \vee (F_\gamma \wedge F_\beta) < (F_\alpha \wedge F_\gamma) \vee (F_\gamma \wedge F_\beta)$ which implies $(F_\alpha \vee F_\gamma) \wedge F_\beta < F_\gamma \wedge (F_\alpha \vee F_\beta)$. Since by theorem 2.21(ii), $F_\alpha \vee F_\gamma = 1_X$ and $F_\alpha \vee F_\beta = 1_X$ which implies $X \wedge F_\beta < F_\gamma \wedge X$ which implies $F_\beta < F_\gamma$. From the definition of fuzzy maximal G#rg-closed set it follows that $F_\beta = F_\gamma$. This is a contradiction to the fact that F_α, F_β and F_γ are different from each other. Therefore $(F_\alpha \wedge F_\beta) \nless (F_\alpha \wedge F_\gamma)$.

Theorem 2.25**:** Let F be a fuzzy maximal G#rg-closed set and x be a fuzzy element of F, then F= ⋁{E: E is a fuzzy G#rg-closed set containing fuzzy elementx_α such that F ⋁ E≠1_X}.

*Proof***:** By theorem 2.24 and from fact that F is a fuzzy G#rg-closed set containing x_α, we have F< ⋁{E: E is a fuzzy G#rg-closed set containing x_α such that F ⋁ E≠ 1_X}< F. Therefore, we have the result.

Vol. 21, No. 1, (2024)
ISSN: 1005-0930

Theorem 2.26**:** If F be a proper nonempty cofinite fuzzy G#rg-closed subset, then there exists (cofinite) fuzzy maximal G#rg-closed set E such that $F < E$.

*Proof***:** If F is a fuzzy maximal G#rg-closed set, we may set E=F. Suppose F is not a fuzzy maximal G#rg-closed set, then there exists (cofinite) fuzzy G#rg-closed set F_1 such that $F< F_1 \neq 1_X$. If F_1 is a fuzzy maximal G#rg-closed set, we may set $E=F_1$. If F_1 is not a fuzzy maximal G#rg-closed set, then there exists a (cofinite) fuzzy G#rg-closed set F_2such that $F< F_1< F_2\neq 1_X$. Continuing this process, we have a sequence of fuzzy G#rg-closed sets, $F <F_1< F_2< F_3< F<\ldots < F_k<\ldots$ .Since F is a cofinite fuzzy set, this process repeats only finitely. Then, finally we get a fuzzy maximal G#rg-closed set $E = E_n$ for some positive integer n.

Theorem 2.27**:** Let F be a fuzzy maximal G#rg-closed set. If x_α is a fuzzy element of $1_X - F$, then $1_X - F < E$ for any fuzzy G#rg-closed set E containing fuzzy element x_α.

*Proof***:** Let F be a fuzzy maximal G#rg-closed set and $x_\alpha \in 1_X - F$. $E < F$ for any fuzzy G#rg-closed set E containing x_α. Then $E \vee F = 1_X$ by theorem 2.21(ii). Therefore $1_X - F < E$.

We now introduce minimal G#rg-closed sets and maximal G#rg-open sets in topological spaces as follows,

Definition 2.28**:** A proper nonempty fuzzy G#rg-closed subset F of fts X is said to be a fuzzy minimal G#rg-closed set if and only if any fuzzy G#rg-closed set which is contained in F is 0_Xor F.

Remark 2.29: Every fuzzy minimal G#rg-closedset need not a fuzzy minimal closed set as seen from the following example.

Example 2.30: Let X = {a, b, c, d, e} with fuzzy subsets 0_x ={(a, 0), (b, 0), (c, 0), (d, 0)} = 0, β_1= {(a, 1)}, β_2= {(d, 1), (e, 1)}, β_3= {(a, 1), (d, 1), (e, 1)}, α_1= {(b, 1), (c, 1)}, α_2= {(a, 1), (b, 1), (c, 1)}, α_3= {(b, 1), (c, 1), (d, 1), (e, 1)} and 1_x ={(a, 1), (b, 1), (c, 1), (d, 1)} = 1 with fuzzy topology oof X is T = {0_X, β_1, β_2, β_3, 1_X}then the fuzzy closed sets in X are0_X, 1_X,α_1, α_2, α_3. Fuzzy minimalclosed sets are α_1= {(b, 1), (c, 1)}. Fuzzy G#rg-closed sets in X are 0_X, 1_X, α_1= {(b, 1), (c, 1)}, α_2= {(a, 1), (b, 1), (c, 1)}, α_4= {(a, 1), (b, 1), (d, 1)}, α_5= {(a, 1), (b, 1), (e, 1)}, α_6= {(a, 1), (c, 1), (d, 1)}, α_7= {(a, 1), (c, 1), (e, 1)}, α_8= {(b, 1), (c, 1), (d, 1)}, α_9= {(b, 1), (c, 1), (e, 1)}, α_{10}= {(a, 1), (b, 1), (c, 1), (d, 1)}, α_{11}= {(a, 1), (b, 1), (c, 1), (e, 1)}, α_{12}= {(a, 1), (b, 1), (d, 1), (e, 1)}, α_{13}= {(a, 1), (c, 1), (d, 1), (e, 1)}, α_3= {(b, 1), (c, 1), (d, 1), (e, 1)}. Fuzzy minimal G#rg-closed sets are α_1= {(b, 1), (c, 1)}, α_4= {(a, 1), (b, 1), (d, 1)}, α_9= {(a, 1), (b, 1), (e, 1)}, α_6= {(a, 1), (c, 1), (d, 1)}, α_7= {(a, 1), (c, 1), (e, 1)}. Here α_4= {(a, 1), (b, 1), (d, 1)}, α_5= {(a, 1), (b, 1), (e, 1)}, α_6= {(a, 1), (c, 1), (d, 1)}, α_7= {(a, 1), (c, 1), (e, 1)} are fuzzy minimal G#rg-closed set but not fuzzy minimal closed set.

Definition 2.31: A proper nonempty fuzzy G#rg-open set U of fts X is said to be a fuzzy maximal G#rg-open set if and only if any fuzzy G#rg-open set which contains U is either 1_X or U.

Remark 2.32: Every fuzzy maximal G#rg-open set need not fuzzy maximal open set as seen from the following example.

Example 2.33: Let X = {a, b, c, d, e} with fuzzy subsets 0_x ={(a, 0), (b, 0), (c, 0), (d, 0)} = 0, β_1= {(a, 1)}, β_2= {(d, 1), (e, 1)}, β_3= {(a, 1), (d, 1), (e, 1)} and 1_x ={(a, 1), (b, 1), (c, 1), (d, 1)} = 1 with fuzzy topology of X is T = {0_X, β_1, β_2, β_3, 1_X}, then fuzzy maximal open sets are β_3= {(a, 1), (d, 1), (e, 1)} and fuzzy G#rg-open sets of X are 1_X, 0_X, β_1={(a, 1)}, α_1= {(b, 1)}, α_2= {(c, 1)}, α_3= {(d, 1)}, α_4= {(e, 1)}, α_5={(a, 1), (d, 1)}, α_6= {(a, 1), (e, 1)}, α_7 = {(b, 1), (d, 1)}, α_8= {(b, 1), (e, 1)}, α_9 ={(c, 1), (d, 1)}, α_{10} = {(c, 1), (e, 1)}, β_2= {(d, 1), (e, 1)}, β_3 = {(a, 1), (d, 1), (e, 1)}, then fuzzy maximal G#rg-open sets are α_7 = {(b, 1), (d, 1)}, α_8= {(b, 1), (e, 1)}, α_9 ={(c, 1), (d, 1)}, α_{10} = {(c, 1), (e, 1)}, β_3 = {(a, 1), (d, 1), (e, 1)}. Butα_7 = {(b, 1), (d, 1)}, α_8= {(b, 1), (e, 1)}, α_9 ={(c, 1), (d, 1)}, α_{10} = {(c, 1), (e, 1)} are fuzzy maximal G#rg-open sets but not fuzzy maximal open sets.

Theorem 2.34: A proper non-empty fuzzy subset A of fts X is a fuzzy maximal G#rg-open set if and only if $1_X - A$ is a fuzzy minimal G#rg-closed set.

Proof: Let A be a fuzzy maximal G#rg-open set. Suppose $1_X - A$ is not a fuzzy minimal G#rg-closed set. Then there exists a fuzzy G#rg-closed set $F \neq 1_X - A$ such that $0_X \neq F < 1_X - A$. That is $A < 1_X - F$ and $1_X - F$ is a fuzzy G#rg-open set. This contradicts our assumption that A is a fuzzy minimal G#rg-closed set.

Conversely let $1_X - A$ be a fuzzy minimal G#rg-closed set. Suppose A is not a fuzzy maximal G#rg-open set. Then there exists a fuzzy G#rg-open set $E \neq A$ such that $A < E \neq 1_X$. That is $0_X \neq 1_X - E < 1_X - A$and $1_X - E$ is a fuzzy G#rg-closed set. This contradicts our assumption that $1_X - A$ is a fuzzy minimal G#rg-closed set. Therefore, A is a fuzzy maximal G#rg-closed set.

Vol. 21, No. 1, (2024)
ISSN: 1005-0930

References:

[1] Azad K. K. On fuzzy semi-continuity, fuzzy almost continuity and fuzzy weakly continuity. J. Math. Anal. Appl. 82 (1981), 14–32.

[2] Chang C. L. Fuzzy topological spaces. J. Math.Anal.Appl. 24 (1968), 182–190.

[3] Ittanagi, B. M.; Wali, R. S. On fuzzy minimal open and fuzzy maximal open sets in fuzzy topological spaces. Int. J. of Mathematical Sciences and Applications, 1(2)(2011), 1023-1037.

[4] Holabasayya Sankannavar and Jenifer J. Karnel, A study of fuzzy generalized #rg-closed sets and their role in fuzzy topological spaces, Communications in Mathematics and Applications, ICRTACM23-049,Nov-2023(Communicated).

[5] Zadeh L. A. Fuzzy sets. Inf Comput., 8 (1965), 338–353.

Vol. 21, No. 1, (2024)
ISSN: 1005-0930