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A METHOD TO PREDICT THE VIBRATION CHARACTERISTICS OF THIN WALLED CIRCULAR CYLINDER

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Abstract

This work presents an approximate way to determining the natural frequencies of thin-walled cylinders. Nonetheless, a unique, simple device based on the virtual work concept has been developed to estimate the intrinsic frequencies and vibration amplitudes without the need for complex numerical resolution. Furthermore, the model's applicability is broadened to encompass all common constraint scenarios. Finding the natural frequencies of a continuous cylinder may be reduced to an eigenvalue problem using a matrix whose components are determined solely by the cylinder's geometrical characteristics, the material's mechanical properties, and known numerical parameters. These are pre-estimated for specific boundary circumstances, such pinned or clamped end limits. The proposed formulation may handle any combination of constraints, but is restricted to the analysis of a pinned-pinned cylinder for the purpose of conciseness. The results of the FEM study were used to assess the reliability of the model. According to these assessments, the maximum inaccuracy in relation to the precise solutions for the lowest natural frequency for all mode forms of the pinned-pinned case is around 3%, offering a superb trade-off between accuracy and usability.

Keywords: *Circular Cylindrical Shell, Virtual work principle, Natural Frequency, Eigenvalue Problem*

1.INTRODUCTION

Researchers are interested in characterizing the vibratory behavior of thin-walled cylinders since shells are widely used as structural components in many engineering applications. In particular, estimating their natural frequency is critical to minimize catastrophic defects throughout the manufacturing process and regular usage, when time-varying pressures often stress these parts.

Thin-walled cylinders' continuous structure makes investigating their free vibrations significantly more challenging than a discrete multi-degree-of-freedom system. The integration of partial differential equations seldom yields an accurate closed-form solution, which is somewhat complex.

Finite element analysis, numerical methods and simplified analytical models [1-20] are more commonly used to reach the resolution. However, the finite element technique (FEM) may necessitate a convergence analysis. On the other hand, the use of simplifying assumptions allows for an analytical solution to the problem at the expense of accuracy, whereas advanced numerical techniques allow for the resolution of exceedingly precise models but can be difficult to program. This innovative model, on the other hand, combines high accuracy with user-friendliness. It draws inspiration from [4], which gave the natural frequencies via an explicit series of algebraic equations that did not require intricate or repetitive numerical solutions. The dynamic equilibrium equations are obtained as displacement functions, beginning with Love's theory for thin-walled cylinders, which has been modified by Reissner and simplified by Donnell's assumptions. Then, Hamilton’s principle concept is used. Similar to Rayleigh's approach, the assumption of suitable eigen functions permits a quick solving process based on the solution of a series of basic algebraic equations. However, depending on the mode shape order, this technique includes two alternative sets of eigen functions and is limited to clamped-clamped cylinders.

A reformulation of [4] is suggested in this study. The constitutive, compatibility, and equilibrium equations are specifically the same. However, the virtual work principle is applied and several eigen functions are first postulated. Furthermore, an eigen value problem is derived from the cascaded algebraic resolutive technique. Faster resolution is achieved using the novel approach, which is easily adaptable to any constraint situation, not only the clamped-clamped one. This publication, for instance, just presents the findings for a pinned-pinned cylinder. The suggested method's dependability is evaluated by contrasting its outcomes with those of the FEM analysis. It has been determined that the model is both effective and efficient with a maximum inaccuracy of 3%.

2. THEORY AND METHODOLOGY

A thin-walled circular cylinder with a finite length (l), constant thickness (h), and mean radius (a) composed of a material with density (ρ), Young's modulus (E), and Poisson's ratio (v) is shown in Fig.1 along with its orthogonal local reference system, which consists of longitudinal direction x, circumferential direction s, and radial direction r.

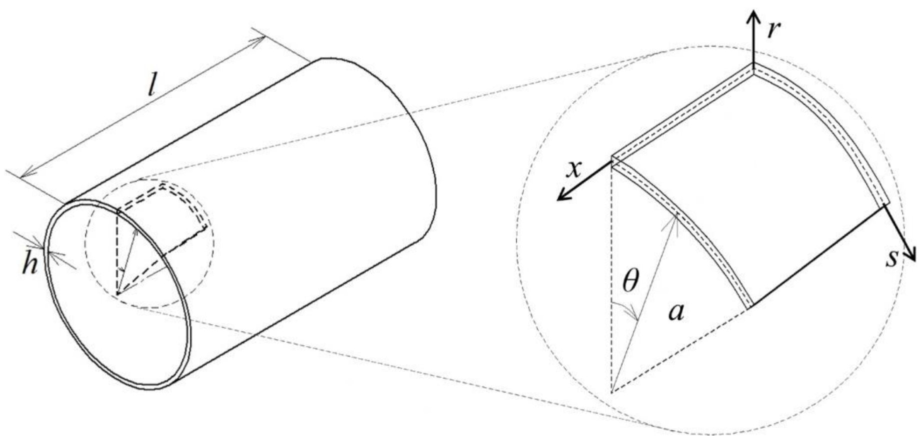

***Fig. 1**. Coordinate System of a Thin-walled Cylinder and its Geometry*

The dynamic equilibrium equations and compatibility equations are constructed using Love's theory [2], which has been modified by Reissner [7] and Donnell's assumptions [11]. The internal forces and moments are then expressed as functions of the deformations using the constitutive equations. Forces and moments are produced as functions of displacements by substituting the compatibility equations into these later equations. After adding these forces and moments to the dynamic equilibrium, the motion equations that follow are obtained:

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$$\left.\begin{aligned}K\left(\frac{\partial^2 u_x}{\partial x^2}+\frac{1-\nu}{2a^2}\frac{\partial^2 u_x}{\partial\theta^2}+\frac{1+\nu}{2a}\frac{\partial^2 u_s}{\partial x\partial\theta}+\frac{\nu}{a}\frac{\partial u_r}{\partial x}\right)=\rho h\frac{\partial^2 u_x}{\partial t^2}\\ K\left(\frac{1}{a^2}\frac{\partial^2 u_s}{\partial\theta^2}+\frac{1-\nu}{2}\frac{\partial^2 u_s}{\partial x^2}+\frac{1+\nu}{2a}\frac{\partial^2 u_x}{\partial x\partial\theta}+\frac{1}{a^2}\frac{\partial u_r}{\partial\theta}\right)=\rho h\frac{\partial^2 u_s}{\partial t^2}\\ K\left[-\frac{\nu}{a}\frac{\partial u_x}{\partial x}-\frac{1}{a^2}\frac{\partial^2 u_s}{\partial\theta}-\frac{u_r}{a^2}-\frac{h^2}{12}\left(\frac{\partial^4 u_r}{\partial x^4}+\frac{1}{a^4}\frac{\partial^4 u_r}{\partial\theta^4}+\frac{2}{a^2}\frac{\partial^4 u_r}{\partial x^2\partial\theta^2}\right)\right]=\rho h\frac{\partial^2 u_r}{\partial t^2}\end{aligned}\right\} \quad (1)$$

$$where\ K=\frac{Eh}{1-\upsilon^2} \quad (2)$$

The mathematical process that yielded (Eq.1) is omitted for the purpose of concision. The virtual work principle (Eq. 3), which states that for any virtual displacements that fulfill the requirements, the virtual work δW of all forces applied to the system, including inertial actions, is zero, is presented using the equations of motion and Hamilton's notion.

$$\delta W=aK\int_0^{2\pi}\int_0^{l}\left\{\left[\frac{\partial^2 u_x}{\partial x^2}+\frac{1-\nu}{2a^2}\frac{\partial^2 u_x}{\partial\theta^2}+\frac{1+\nu}{2a}\frac{\partial^2 u_s}{\partial x\partial\theta}+\frac{\nu}{a}\frac{\partial u_r}{\partial x}-\rho h\frac{\partial^2 u_x}{\partial t^2}\right]\delta u_x+\left[\frac{1}{a^2}\frac{\partial^2 u_s}{\partial\theta^2}+\frac{1-\nu}{2}\frac{\partial^2 u_s}{\partial x^2}+\frac{1+\nu}{2a}\frac{\partial^2 u_x}{\partial x\partial\theta}+\frac{1}{a^2}\frac{\partial u_r}{\partial\theta}-\rho h\frac{\partial^2 u_s}{\partial t^2}\right]\delta u_x+\left[-\frac{\nu}{a}\frac{\partial u_x}{\partial x}-\frac{1}{a^2}\frac{\partial^2 u_s}{\partial\theta}-\frac{u_r}{a^2}-\frac{h^2}{12}\left(\frac{\partial^4 u_r}{\partial x^4}+\frac{1}{a^4}\frac{\partial^4 u_r}{\partial\theta^4}+\frac{2}{a^2}\frac{\partial^4 u_r}{\partial x^2\partial\theta^2}\right)-\frac{1-\upsilon^2}{E}\rho\frac{\partial^2 u_r}{\partial t^2}\right]\delta u_r\right\}dxd\theta=0 \quad (3)$$

Since $\delta u_x, \delta u_s and\ \delta u_r$ are arbitrary virtual displacements, (Eq.3) is only valid if each of the three addends is null.

As in [4], adopting appropriate displacements u_x, u_s, and u_r as eigen functions of the issue allows for a more straightforward mathematical approximation approach to the problem of free vibrations of cylindrical shells. The free vibrations of a thin-walled circular cylinder consist of m longitudinal half-waves and n circumferential waves. Consequently, each mode form is characterized by a pair of m and n values. Consequently, each mode form is characterized by a pair of n and m values. Circumferential waves are unaffected by boundary conditions, in contrast to longitudinal half-waves, which depend on them. This is similar to the transverse vibrations of beams that are limited by the same restrictions. As such, the same hypotheses that were put forward in [4], carefully chosen to ensure orthogonality, are also considered.

$$\left.\begin{aligned}u_x&=A_x\frac{d}{dx}f_r(x)\cos(n\theta)\cos(\omega t)\\ u_s&=A_s f_r(x)\sin(n\theta)\cos(\omega t)\\ u_r&=A_r f_r(x)\cos(n\theta)\cos(\omega t)\end{aligned}\right\} \quad (4)$$

where $f_r(x)$ is the eigen function of the beam subjected to the same constraints of the cylinder under analysis. For example, for a pinned-pinned beam, $f_r(x)=sin\beta_i l\frac{x}{l}$, where $\beta_i\ l$ are the roots of the related frequency equation $\sin\beta_i l=0$.

All boundary conditions may be applied to the following formulation, though, because it is quite broad.

A system of three equations is produced by taking into consideration (Eq. 4)and normalizing (Eq. 3) with the cylinder length l.

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$$\left.\begin{aligned}
&= \int_0^1 \left[\left(\frac{\partial^2 u_x}{\partial x^2} + \frac{1-\nu}{2a^2}\frac{\partial^2 u_x}{\partial \theta^2} + \frac{1+\nu}{2a}\frac{\partial^2 u_s}{\partial x \partial \theta} + \frac{\nu}{a}\frac{\partial u_r}{\partial x} + \Delta u_x \right) \delta u_x \right] dX = 0 \\
&\int_0^1 \left[\left(\frac{1}{a^2}\frac{\partial^2 u_s}{\partial \theta^2} + \frac{1-\nu}{2}\frac{\partial^2 u_s}{\partial x^2} + \frac{1+\nu}{2a}\frac{\partial^2 u_x}{\partial x \partial \theta} + \frac{1}{a^2}\frac{\partial u_r}{\partial \theta} + \Delta u_s \right) \delta u_x \right] dX = 0 \\
&\int_0^1 \left\{ \left[\frac{\nu}{a}\frac{\partial u_x}{\partial x} + \frac{1}{a^2}\frac{\partial^2 u_s}{\partial \theta} + \frac{u_r}{a^2} + \frac{\eta^2}{12}\left(\frac{\partial^4 u_r}{\partial x^4} + \frac{1}{a^4}\frac{\partial^4 u_r}{\partial \theta^4} + \frac{2}{a^2}\frac{\partial^4 u_r}{\partial x^2 \partial \theta^2} \right) - \Delta u_r \right] \delta u_r \right\} dX = 0
\end{aligned}\right\} \quad (5)$$

The subsequent matrix formulation is obtained by substituting (Eq. 4) in (Eq. 5)

$$\left(\bar{\bar{D}} - \Delta \bar{\bar{I}}\right)\{A\} = \{0\} \quad (6)$$

where $\bar{\bar{D}}$ is the following matrix, $\bar{\bar{I}}$ is the identity matrix and $\{A\} = \{A_x;\ A_s;\ A_r\}$ is the unknown vector containing the displacements amplitudes in thethree directions,

$$\bar{\bar{D}} = \begin{bmatrix} -\frac{I_{13}}{I_{11}} + \frac{1-V}{2\alpha^2}n^2 & -n\frac{1+V}{2\alpha} & -\frac{V}{\alpha} \\ \frac{1+V}{2\alpha}n\frac{I_{02}}{I_{00}} & -\frac{1+V}{2}\frac{I_{02}}{I_{00}} + \frac{n^2}{\alpha^2} & \frac{n^2}{\alpha^2} \\ \frac{V}{\alpha}\frac{I_{02}}{I_{00}} & \frac{n}{\alpha^2} & \frac{1}{\alpha^2} + \frac{\eta^2}{12}\left(\frac{I_{04}}{I_{00}} + \frac{n^4}{\alpha^4} - \frac{2}{\alpha^2}n^2\frac{I_{02}}{I_{00}}\right) \end{bmatrix} \quad (7)$$

$$where\ I_{13} = \int_0^1 f_r^1(X) f_r^3(X) dX,\ \ I_{11} = \int_0^1 f_r^1(X) f_r^1(X) dX \quad (8)$$

$$I_{02} = \int_0^1 f_r(X) f_r^2(X) dX\ , I_{00} = \int_0^1 f_r(X) f_r(X) dX \quad (9)$$

$$I_{04} = \int_0^1 f_r(X) f_r^4(X) dX \quad (10)$$

$f_r^k(X)$ is the k[th] order derivative of $f_r(X)$. (11)

It is possible to estimate these integrals a priori without knowing the actual cylinder dimension because of the normalizing by the cylinder length. They may therefore be used to analyze the free vibrations of any cylinder that is subject to the same restrictions after only having to be computed once for every constraint condition.

Finally, as demonstrated by (Eq. 6), the natural frequency of any thin-walled cylinder may be computed with ease by working out the eigen values problem of the matrix $\bar{\bar{D}}$. The natural frequency may be derived using the three eigenvalues Δ1, Δ2 and Δ3 in the following way,

$$f_i = \frac{1}{2\pi}\sqrt{\frac{E\Delta_i}{(1-\nu^2)\rho l^2}} \qquad \text{for i=1, 2,3} \quad (12)$$

In summary, the eigen function defining the transverse free vibrations of a beam exposed to the same restrictions as the cylinder is first identified in the suggested analysis of free vibrations of cylindrical shells. The matrix $\bar{\bar{D}}$ is then calculated for a pair of values for m and n. The matrix $\bar{\bar{D}}$ eigen values are used to calculate the natural frequencies and its eigenvectors include the displacement and amplitude ratios of each modeshape.

3. RESULTS AND DISCUSSION

For a pinned-pinned cylinder with the following characteristics: a = 76 mm, *l*=305 mm, h = 0.254 mm, ρ = 7833 kg/m^3, E= 207 kN/mm^2, and v = 0.3. *Table 1* displays the frequencies f_1, f_2 and f_3 are listed for m < 4 and n ≤ 10. They are adequate to see that f_1 is less than f_2 and f_3 by

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an order of magnitude. As a result, the frequency is associated with the largest risk of redundancy. Furthermore, $f1$ has a minimum for fixed m, which happens for a greater value of n as m grows, whereas $f2$ and $f3$ are monotonically growing by both n and m.

***Table 1**: Natural frequencies for m < 4 and n ≤ 10.*

n	***m*=1**			***m*=2**			***m*=3**		
	***f*1(Hz)**	***f*2(Hz)**	***f*3(Hz)**	***f*1(Hz)**	***f*2(Hz)**	***f*3(Hz)**	***f*1(Hz)**	***f*2(Hz)**	***f*3(Hz)**
1	2886	10009	17209	6445	14059	21944	8556	17890	29228
2	1265	14889	26403	3677	18061	29947	5821	21647	35508
3	656	20923	36632	2178	23194	39414	3896	26222	43820
4	423	27319	47298	1396	29012	49558	2681	31491	53176
5	372	33847	58183	985	35177	60072	1931	37225	63123
6	431	40433	69190	792	41523	70807	1477	43249	73434
7	551	47048	80272	751	47971	81681	1227	49455	83983
8	705	53681	91403	819	54481	92649	1132	55780	94694
9	887	60325	102566	956	61031	103683	1158	62184	105520
10	1092	66976	113753	1139	67607	114764	1272	68644	116431

Table 2 shows the amplitude ratios only for n = 4 and m ≤ 3. While for other combinations of m and n it provides comparable results. The amplitude at the lowest natural frequency $f1$ is Ar, (ratios less than unity) indicating primarily radial motion. Similarly, at frequencies $f2$ and $f3$, Ax and As predominate, resulting in longitudinal and circumferential modes.

***Table 2:** Amplitude ratios for m ≤ 3 and n = 4.*

	***m* = 1**		***m* = 2**		***m* = 3**	
	A_x/A_r	A_s/A_r	A_x/A_r	A_s/A_r	A_x/A_r	A_s/A_r
f1	0.014	0.252	0.011	0.255	0.008	0.249
f2	3.697	1.897	1.020	2.155	0.521	2.561
f3	0.240	4.097	0.259	4.380	0.288	4.822

The result with those from FEM study was done in Ansys. In analysis SHELL181 linear elements are selected and around 36433 elements are observed to analyze a thin-walled circular cylinder. The errors of all theories with respect to the experiment are also shown in Table 3.

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Table3: *Present model results and FEM results along with errors between them*

	***m*=1**			***m*=2**			***m*=3**			***m*=4**		
n	***f*1(Hz)**		**Error**	***f*1(Hz)**		**Error**	***f*1(Hz)**		**Error**	***f*1(Hz)**		**Error**
	Present	**FEM**	**(%)**	**Present**	**FEM**	**(%)**	**Present**	**FEM**	**(%)**	**Present**	**FEM**	**(%)**
1	2885.9	2886	−0.0004	6444.7	6445	−0.010	8555.7	8557	−0.018	9527.5	9530	−0.021
2	1265.1	1265	0.011	3676.6	3678	−0.025	5820.8	5823	−0.043	7344.4	7349	−0.055
3	656	654.8	0.19	2178.1	2179	−0.034	3896.1	3899	−0.071	5413	5418	−0.096
4	422.7	418.7	0.96	1396.4	1396	0.0093	2681.1	2684	−0.092	3986.1	3992	−0.14
5	372	364.7	2.01	985.3	983.5	0.18	1931.4	1933	−0.082	2994.2	2999	−0.17
6	431.4	422.6	2.07	792.1	788.1	0.51	1477.2	1477	−0.013	2322.6	2327	−0.18
7	550.6	542.2	1.56	751.4	745.7	0.76	1226.6	1225	0.12	1884.4	1887	−0.15
8	705.3	698.2	1.02	818.6	812.7	0.72	1131.6	1129	0.23	1626.4	1628	−0.10
9	886.7	881.8	0.56	955.9	951.5	0.47	1157.8	1155	0.21	1515.7	1517	−0.072
10	1091.9	1090	0.17	1138.9	1137	0.15	1272.5	1272	0.048	1525.7	1528	−0.12
11	1319.7	1322	−0.18	1355.2	1358	−0.18	1448.9	1452	−0.20	1629.6	1634	−0.28
12	1569.7	1578	−0.51	1599	1607	−0.50	1669.5	1678	−0.49	1803	1812	−0.50

A relative error is defined as follows:

$$Error_{\omega} = \frac{\omega_{present} - \omega_{ansys}}{\omega_{present}} \times 100\% \qquad (13)$$

The correctness of the results was evaluated, with a maximum error of 2.07% for m = 1 and n = 6 as observed in Table 3

There is strong consistency between the FEM results and those obtained by the proposed model, which it can be observed in Fig2. For the sake of simplicity, the comparison is made for frequency $f1$s versus the value of n for fixed m=1,2,3 since $f1$ is considered as it is lowest compared to $f2$ and $f3$.

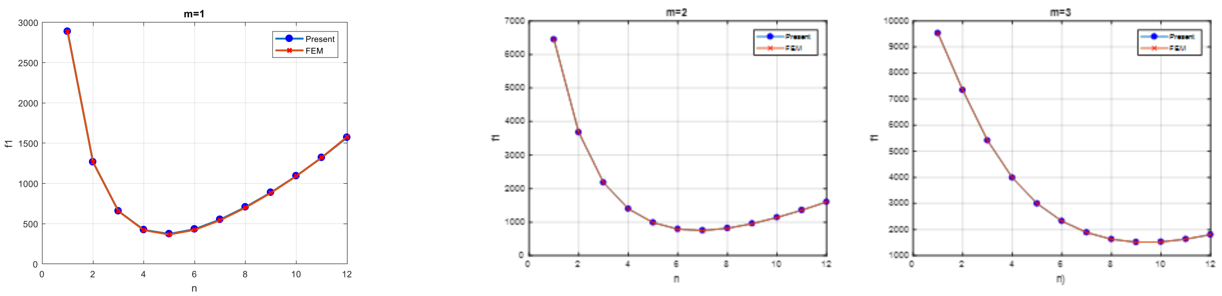

Figure 2. Comparison between the Natural frequency $f1$ (obtained by the proposed model and FEM results) and n for m=1,2,3.

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4. CONCLUSION

This study created a novel method to study the free vibrations of an isotropic circular cylindrical shell under different boundary conditions. The cylinder equations of motion of Donnell-Mushtari's shell theory were introduced in the principle of virtual work. The eigenvalue problem was the only remaining component of the greatly simplified mathematical approach. The eigen function of a beam as constrained as the cylinder was incorporated, and the resulting system was normalized to the cylinder length. The findings for a pinned-pinned cylindrical shell were contrasted with a FEM research that produced a maximum inaccuracy of 3%. Because it combines excellent accuracy and ease of use, the novel model is ideal for performing exploratory research on the resonance state of shell structures. It may be extended to any boundary conditions as well. Pinted, free end, and clamped cylinders can be handled with a specific formulation. The accuracy of the model will be assessed in subsequent studies using various combinations of constraints.

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